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Final Report UD-8

CROSSTIE LIFE REDUCTION AS A FUNCTION OF LOSS OF ADJACENT TIES SUPPORT

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EXECUTIVE SUMMARY

Railroad cross-ties (sleepers) are a key component of the track structure and play an important role in the distribution of train loading through the track. Automated cross-tie inspections, which are becoming increasingly significant in the inspection of the cross-ties, are important in planning and optimizing tie replacement. Furthermore, the data these inspections provide on tie condition enable maintenance engineers to better understand the behavior of the ties and their associated life. By using inspection data taken from the same track in different years, it is possible to develop improved tie life models that take into account local conditions. Using these different tie conditions, and the corresponding different periods in the lifespan of a tie, it is possible to determine average tie life using mathematical modeling techniques, such as piecewise reconstruction. It is also possible to develop a model that shows how the probability of tie failure grows over time and changes depending on the loss of adjacent support.

The dataset used consists of tie inspection data for inspections carried out on the same track during the period 2016 to 2019. Ties are grouped based on their adjacent tie condition. This report provides different methods to predict and model tie life based on support condition, as defined by the condition of adjacent cross-ties. The analysis approaches are based on the use of tie condition data from two different inspections performed over a span of 3 years.

A piecewise reconstruction of the average tie life was performed and used to compare the tie degradations rates with respect to loss of adjacent tie support. The first method used to reconstruct an average tie life was using regression. Regression functions were developed based on the distributions of the different tie score transitions from 2016 to 2019 in different support groups. These functions were then used recursively to predict the tie score change over time. Dijkstra's algorithm was then applied to model each group's average tie life. In a third analysis approach, Markov chains were used for the determination of the probability of tie failure as a function of loss of support.

The results show different average tie lives for different support conditions and confirms the fact that loss of support contributes significantly to premature tie failure. A life reduction formula was then generated based on the three analysis approaches.

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1. Introduction and Overview

Railroad cross-ties (sleepers[1]) are a key component of the track structure and play an important role in the distribution of train loading through the track. Tie inspections, which identify the condition of individual cross-ties, represent a major input into track maintenance operations in the railroad industry. These inspections provide engineers and technicians with critical data allowing them to develop an appropriate and efficient tie maintenance and replacement plan, to include key maintenance actions such as spot tie replacement and large-scale tie replacement using mechanized production gangs [1, 2, 3]. For instance, as a safety measure, tie replacement should be performed before ties fail, i.e., are defined to be defective by the Federal Railroad Administration [4, 5]. One of the most important phases of track maintenance is the inspection phase, which for cross-ties involves identifying the ties to be replaced. Ineffective tie replacement is not only expensive, but can also be dangerous in the long-term [1] while an effective tie replacement strategy, based on advanced inspection, can save tens of millions of dollars annually [7]. The new generation of automated cross-tie inspections, which are playing an increasingly important role in the inspection of the cross-ties, is becoming more and more important in planning and optimizing tie replacements and corresponding maintenance. Furthermore, the data they provide on tie condition enable maintenance engineers to better understand the behavior of the ties and their associated life.

Although automated track and tie inspections are crucial, their frequency depends on many conditions, to include failure rate, economics, available budgets and, of course, track safety considerations [8]. Because the inspections are not always performed annually, drawing conclusions from the limited number of data to make predictions and planning decisions can be challenging. One way to overcome this problem is to use inspection data to predict a tie's life, and to use that predicted tie life to optimize the maintenance processes. One such study suggests that the condition of the ties adjacent to the study tie impacts its life directly with different adjacent tie support conditions resulting in different tie lives [9].

The intent of this research is to:

- Model a tie's life based on the adjacent tie condition,
- Predict the probability of a tie changing condition within a time period based on its support condition,
- Help forecast a tie's remaining life as a function of loss of adjacent ties support,
- Help make better decisions on tie replacement and tie gang prioritization,
- Contribute to the improvement of railroad infrastructure Reliability, Availability, Maintainability, and Safety (RAMS)

I.1. Factors Influencing Tie Failure and Tie Life

Average tie life has been modelled in different ways to include statistical forecasting models, empirical models, and mechanistic models [10, 11]. Two major modelling approaches were suggested in previous studies: A statistical tie life approach that predicts the actual number of failed ties each year and an "Average" tie life modelling approach [10, 11]. Various analyses of

[1] Railroad cross-ties are often referred to as sleepers.

average wood cross-tie lifespan show a range of wood tie life of the order of 25 to 40 years depending on climate, treatment, track and traffic conditions [12, 13, 14]. Tie failure mechanisms can include environmental decay, mechanical deterioration and damage (burnt, broken) [6]. Further subdividing tie failure mechanisms shows that tie life is affected by track and operating factors (curvature, traffic density, axle load, grade etc.[15]), climate factors (temperature, water, moisture), biological factors (fungi), incompatibility factors (physical and chemical degradation), use factors (traffic, maintenance, track geometry, and accidents), stress factors (abrasion and compression due to ballast, and load factors (impact compression and impact bending due to vertical loads, and spike loading due to lateral loads), [11].

One key factor that has been discussed as affecting tie life is support condition, often defined as track support condition or track modulus [1, 16]. While the effect of support condition on tie life has been difficult to quantify, a recent study attempted to examine the effect of the support condition defined by the condition of adjacent cross-ties, using Beam on Elastic Foundation theory [9]. This study showed that the support conditions associated with the condition of the adjacent cross-ties do in-fact contribute to premature tie failure [2]. The work presented in this paper builds upon this earlier research to extend its results, allowing for the calculation of actual tie life, as well as allowing for a more effective analysis approach using more advanced data analytics.

I.2. Support Condition

As noted, support condition, and in particular the condition of adjacent ties can and do affect the life of a cross-tie. This is because in normal track, the dynamic wheel load of a passing train is supported not just by the tie immediately under the wheel but also by the ties adjacent to that center tie under the wheel. This is illustrated in Figure 1, which shows the distribution of wheel load on the tie under the wheel (in red) and two ties on either side of the center tie. This distribution, which is determined using Beam On Elastic Foundation (BOEF) theory [16], is based on a track modulus of 28 MPa [4,000 lb/in/in] as presented in Reference 9.

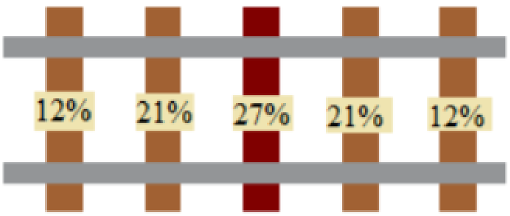


Figure 1: Percent of load carried by each adjacent tie [9]

Thus, support condition can be affected by the loss of support of an adjacent tie if that tie has failed and no longer can carry its share of the vertical wheel load. This, in turn, results in additional load on the tie under the wheel. This is discussed in further detail in Reference 9, which showed that the presence of failed adjacent ties, and the associated loss of support, results in a more rapid degradation of the cross-tie and a corresponding shorted tie life. This behavior was defined by Equation 1 below [9]:

Life Reduction coefficient = $1.444\ LS^2 - 1.322\ LS + 0.9931$ Equation 1 [9]

Where the Life Reduction Coefficient is the reduced life of the center tie and *LS* is the loss of support associated with the failed adjacent ties. The higher the loss of support, the shorter the tie life.

Examination of this equation suggests a rather severe effect of loss of support on tie life. The analysis approach also did not provide a mechanism for calculating average tie life, but rather relies on external data for that life value. This research project will build upon the research presented in Reference 9, which used simple regression modeling, and develop a more accurate degradation equation as well as address the issue of projected tie life itself.

I.3. Approach

The analyses presented here-in consists of several different analysis approaches that used not just the simple 1 to 4 tie condition score presented in Reference 9, but rather made us of decimal scores, which provided more detailed tie condition information, and allowed for the use of more detailed and granular modeling process.

The approach presented in this report consists of the following steps:

- Data identification and preparation,
- Analysis of decimal tie scores,
- Development of a model that predicts the reduction in life of a tie as a function of adjacent tie support condition using Dijkstra's Algorithm,
- Development of a model that predicts the change of failure probability over time as a function of adjacent tie support condition using Markov Chains,
- Comparison of different life reduction models,

This will be discussed in detail in this report.

2. Data Sources, Description, and Pre-Processing

The primary data used in this analysis was tie condition data provided by GREX[2] . This tie condition data included data from both the Aurora tie surface inspection system and the Aurora X[i] which also included internal tie condition data from backscatter X-Ray. The provided data also included detailed location of each individual tie, and tie characteristic data. The specific elements of data are discussed below
Three years of inspection data were provided as follows:

- 2016: 10 text files
- 2017: 8 text files
- 2019:8 text files

The dataset consists of tie condition data collected in years 2016, 2017 and 2019 from the same track segment with an overall length of 65 miles. All railroad location and customer information were removed to ensure the data remained anonymous. Table 1 below summarizes the number of observations in each file:

Table 1: Number of observations in each file

	File1	**File2**	**File3**	**File4**	**File5**	**File6**	**File7**	**File8**	**File9**	**File10**	**Total number of ties**
2016	37534	20953	31956	18126	4548	5279	26784	24309	33408	6605	209502
2017	19792	33014	24656	29970	32050	31311	32231	6105	-	-	209129
2019	13000	32495	31859	32052	6014	26252	32193	35527	-	-	209392

A screen shot of a file can be seen in Figure 2:

[2] Georgetown Railway Equipment Company a subsidiary of Loram, Inc.

Tie Num	Tie Start	Tie End S	Mile Pos	Crossing	Switch	Bridge	Guard R	Is Concr	Tie to Ti	Cumulati	Plate Cut	Plate Cut	Plate Cut	Plate Cut	LRail Dif	RRail Dif	Joint Bar	Left Plat	Right Pl	WidthLe	WidthRi	TieLengt	SkewAng	Left Fiel	Left Gag	Right Ga	Right Fi	Tie Scor	TQI Weig	Ballast C	GPS QoS	Curvatur	Ungrade	Internal T	Aurora X	Tie State
0	245	418	212.01	0	0	0	0	0	0	1.16	0.1752	0.1752	0.0876	0	0.0876	0.1752	0	1	5	114	114	8.5	2.507	0	0	0	0	2	2	0.64	5	-0.24	0	-1	2	1
1	513	621	212	0	0	0	0	0	17.61	2.62	0	0.1752	0.0876	-0.088	0.1752	0.1752	0	1	5	104	104	8.9	0.161	0	0	0	0	1	1	0.11	5	-0.23	0	-1	1	1
2	740	855	212	0	0	0	0	0	17.23	4.05	0	0.0876	0	0	0.0876	0	0	1	1	115	115	8.4	0	0	0	0	0	1	1	0.34	5	-0.23	0	-1	1	1
3	953	1103	212	0	0	0	0	0	17.23	5.48	0.0876	0.1752	0.0876	0.0876	0.0876	0.0876	0	1	1	110	110	9	-1.61	0	0	0	0	1	1	0.23	5	-0.22	0	-1	1	1
4	1204	1325	212	0	0	0	0	0	17.68	6.94	0	0.1752	0.1752	0.0876	0.1752	0.1752	0	1	1	106	105	8.5	0.66	0	0	0	0	1	1	0.03	5	-0.22	0	-1	1	1
5	1514	1635	212	0	0	0	0	0	23.18	8.84	0.438	0.3504	0.1752	0.1752	0.3504	0.0876	0	1	1	112	111	8.5	-0.404	0	0	0	0	2	2.1	0.09	5	-0.21	0	-1	2.1	1
6	1812	1927	212	0	0	0	0	0	22.05	10.67	-99	0.1752	0	0	0.1752	0.1752	0	1	1	115	115	8.4	0	0	0	0	0	1	1	3.92	5	-0.21	0	-1	1	1
7	2075	2194	212	0	0	0	0	0	19.81	12.3	0.0876	-99	0.0876	-0.088	0	0.1752	0	1	1	114	114	8.4	-0.214	0	0	0	0	1	1	0.07	5	-0.2	0	-1	1	1
8	2411	2518	212	0	0	0	0	0	24.67	14.34	0.0876	0	0.1752	0.0876	0.0876	0.1752	0	1	1	106	106	8.8	-0.041	0	0	0	0	1	1.2	0.17	5	-0.19	0	-1	1.2	1
9	2650	2769	212	0	0	0	0	0	18.32	15.86	0.2628	0.2628	0.0876	0.1752	0.0876	0.1752	0	1	1	116	116	8.5	0.128	0	0	0	0	2	2.2	0.02	5	-0.19	0	1.8	2.2	1
10	2881	3042	212	0	0	0	0	0	18.84	17.41	0.3504	0.6131	0.2628	0	0.5255	0.2628	0	1	1	114	115	8.5	1.979	0	0	0	0	3	2.9	0.1	5	-0.19	0	1.4	2.9	1
11	3134	3253	212	0	0	0	0	0	17.34	18.84	0.6131	0.8759	0.438	0.2628	0.438	0.3504	0	1	1	109	109	8.5	0.423	0	0	0	0	3	3	0.03	5	-0.18	0	1.8	3	1
12	3372	3499	212	0	0	0	0	0	18.09	20.33	0.0876	0.1752	0.2628	0.1752	0.1752	0.2628	0	1	1	118	118	8.9	-0.365	0	0	0	0	2	1.8	0.05	5	-0.18	0	1.2	1.8	1
13	3618	3740	212	0	0	0	0	0	18.2	21.83	-0.088	0.0876	0	0	0.1752	0.0876	0	1	1	115	116	8.4	0.278	0	0	0	0	1	1	0.05	5	-0.18	0	1.4	1.4	1
14	3886	4003	212	0	0	0	0	0	19.85	23.47	0.0876	0.1752	-99	0	0.0876	0	0	1	1	113	113	9	0.161	0	0	0	0	1	1	0.01	5	-0.18	0	1	1	1
15	4122	4245	212	0	0	0	0	0	17.87	24.95	0.0876	0	0	0	0.0876	0	0	1	1	118	118	8.4	0.214	0	0	0	0	1	1	0.07	5	-0.18	0	2.1	2.1	1
16	4439	4557	212	0	0	0	0	0	23.51	26.89	0.0876	0.1752	0.1752	0.1752	0.1752	0.0876	0	1	1	113	113	8.8	-0.204	0	0	0	0	1	1.4	0.33	5	-0.17	0	1.3	1.4	1
17	4685	4819	212	0	0	0	0	0	18.99	28.45	0.0876	0.3504	0.2628	0.0876	0.3504	0.1752	0	1	1	110	109	8.4	-1.048	0	0	0	0	2	1.6	0.24	5	-0.17	0	1.1	1.6	1
18	4937	5058	212	0	0	0	0	0	18.35	29.97	0.1752	0.2628	0.1752	0.1752	0.0876	0	0	1	1	117	117	8.4	-0.171	0	0	0	0	1	1	0.51	5	-0.17	0	1	1	1
19	5225	5339	212	0	0	0	0	0	21.27	31.73	0	0.3504	0.438	0.0876	0.3504	0.438	0	1	1	106	106	8.9	-0.325	0	0	0	0	3	2.8	0.25	5	-0.17	0	1.7	2.8	1
20	5471	5590	212	0	0	0	0	0	18.58	33.26	0.1752	0.5255	0.5255	0.2628	0.438	0.3504	0	1	1	114	114	8.5	-0.213	0	0	0	0	2	2.2	0.02	5	-0.17	0	1	2.2	1
21	5705	5827	212	0	0	0	0	0	17.61	34.71	0	0.0876	0.0876	0	0.0876	0.0876	0	1	1	108	109	8.5	-0.574	0	0	0	0	1	1	0.22	5	-0.17	0	1.4	1.4	1
22	5963	6082	212	0	0	0	0	0	19.18	36.29	0.1752	0.3504	0.1752	0.0876	0.2628	0.2628	0	1	1	112	111	8.5	-0.32	0	0	0	0	2	2	0.01	5	-0.17	0	1.2	2	1
23	6250	6346	212	0	0	0	0	0	20.6	37.99	0.2628	0.438	0.5255	0.2628	0.2628	0.2628	0	1	1	96	96	7.3	0	0	0	0	0	3	2.7	3.79	5	-0.17	0	1.1	2.7	1
24	6508	6633	212	0	0	0	0	0	20.37	39.66	0.0876	0.3504	0.2628	0.0876	0.2628	0.2628	0	1	1	118	118	8.5	-0.298	0	0	0	0	2	1.5	0.02	5	-0.17	0	1.4	1.5	1
25	6776	6899	212	0	0	0	0	0	19.96	41.31	0.1752	0.3504	0.1752	0.1752	0.2628	0	0	1	1	117	117	8.9	-0.243	0	0	0	0	2	2.4	0.03	5	-0.16	0	1.6	2.4	1
26	7036	7164	212	0	0	0	0	0	19.62	42.93	0.1752	0.6131	0.7007	0.3504	0.5255	0.6131	0	1	1	118	117	8.5	-0.447	0	0	0	0	3	2.8	0.03	5	-0.16	0	3.8	3.8	1
27	7308	7424	212	0	0	0	0	0	19.89	44.57	0.1752	0.3504	0.2628	0	0.1752	0.3504	0	1	1	114	114	9	-0.08	0	0	0	0	2	2.2	0	5	-0.16	0	1.1	2.2	1
28	7529	7707	212	0	0	0	0	0	18.84	46.13	0.2628	0.2628	0.5255	0.1752	0.1752	0.438	1	1	1	113	114	8.5	2.749	0	0	0	0	3	2.6	0.06	5	-0.16	0	1.2	2.6	1
29	7824	7941	212	0	0	0	0	0	19.77	47.75	0.0876	0.1752	0.1752	0	0.1752	0.1752	1	1	1	112	113	8.9	-0.182	0	0	0	0	1	1	0.02	5	-0.16	0	1.1	1.1	1
30	8108	8236	212	0	0	0	0	0	21.64	49.54	0.5255	-99	0.5255	0.2628	-99	0.3504	0	1	1	119	119	8.4	-0.385	0	-1	0	0	3	3	0.15	5	-0.16	0	3	3	1
31	8354	8484	212	0	0	0	0	0	18.47	51.05	0.1752	0.6131	0.2628	0	0.6131	0.2628	0	1	1	115	115	8.4	-0.641	0	0	0	0	3	2.9	1.02	5	-0.16	0	3.5	3.5	1
32	8589	8701	212	0	0	0	0	0	16.9	52.45	0	0.0876	0.0876	0	0.0876	0.0876	0	1	1	111	112	8.4	0.021	0	0	0	0	1	1	0.03	5	-0.15	0	1.4	1.4	1
33	8881	8998	212	0	0	0	0	0	22.02	54.26	0.0876	0.2628	0.2628	0.1752	0.2628	0.1752	0	1	1	110	110	9	-0.28	0	0	0	0	1	1	0.02	5	-0.14	0	1.1	1.1	1
34	9197	9344	211.99	0	0	0	0	0	24.75	56.31	0	0.0876	0	0	0.0876	0.0876	0	1	1	108	108	8.4	1.665	0	0	0	0	1	1	0.09	5	-0.13	0	1.1	1.1	1
35	9428	9546	211.99	0	0	0	0	0	16.19	57.65	0	0.1752	0.1752	0	0.2628	0.1752	0	1	1	109	110	8.6	-0.354	0	0.53	0	0	2	2	0.04	5	-0.12	0	1.2	1.2	1

Figure 2: Screenshot of a file

For each individual tie observation, there are 36 descriptive variables unique to that tie as follows:

- Tie Num or Tie Number which represents the individual tie number, in sequence, i.e., a tie counter which resets for every file
- Tie Start/End Slice: which is the location on the Aurora viewer of the start or end of a tie. In other words, when the data is collected on the field, a 2D height profile of the tie is collected, also known as a slice. When all these slices are put together, they make a 3D representation which is then used to define the condition of the tie as can be seen in Figure 3 below:

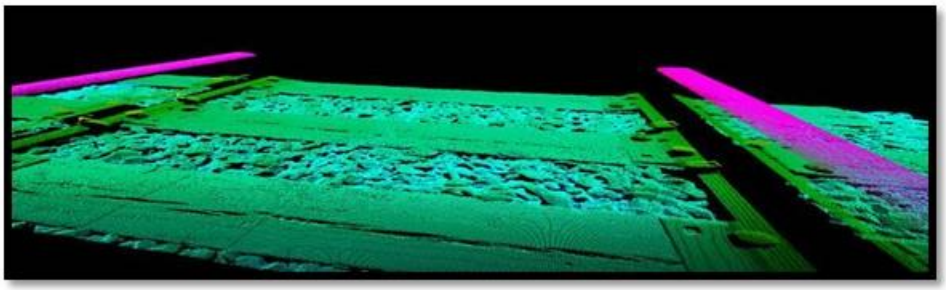


Figure 3: 3D Representation on the Aurora Viewer

The delta between the two columns represent the total number of slices that make up the specific tie.

- Mile Post: which is the mile post location according to customer GPS/MP lapping file
- Crossing: Whether the tie is located in a grade crossing
  - 2: tie is part of a crossing
  - 1: tie is 1 tie from crossing on either direction
  - 0: tie in not in a crossing
- Switch: Whether the tie is located in a turnout.
  - 2: tie is part of a switch,
  - 1: tie is 1 tie from switch on either direction,
  - 0: tie in not in a switch
- Bridge: Whether the tie is located on a bridge.
  - 2: tie is part of a bridge,
  - 1: tie is 1 tie from bridge on either direction,
  - 0: tie in not in a bridge
- Guard Rail: Whether the tie is supporting a guard rail
  - 1: tie is supporting guard rail
  - 0: tie is not supporting guard rail
- Is Tie Concrete or Wood:
  - 1: tie was identified as concrete
  - 0: tie was identified as wood
- Tie to Tie Distance: The distance from tie center to tie center in inches
- Cumulative Distance: The total distance from the start of a collection covered during a collection in feet. It resets for every file.

- Plate Cut LL Max (in): plate cut value for left field side of tie
- Plate Cut LR Max (in): plate cut value for left gage side of tie
- Plate Cut RL Max (in): plate cut value for right field side of tie
- Plate Cut RR Max (in): plate cut value for right gage side of tie
- LRail Diff Plate Cut: plate cut differential for left rail in inches.
- RRail Diff Plate Cut: plate cut differential for right rail in inches
- Joint Bar Tie
    - 1: tie is under a joint bar
    - 0: tie is not under a joint bar
- Left Plate Type: plate type on left rail - Right Plate Type: plate type on right rail,
    - 0: unknown
    - 1: spike plate
    - 2: e-clip
    - 3: Pandrol fast clip
    - 4: spike plate 18 in
    - 5: spike plate 10 in
    - 6: victor plate
- Width Left Side: tie width on left rail side in pixels
- Width Right Side: tie width on right rail side in pixels
- Tie Length Feet: length of a tie is feet
- Skew Angle Degrees: tie skew angle in degrees
- Left Field Adze Depth Inches: depth of adzing on left field side
- Left Gage Adze Depth Inches: depth of adzing on left gage side
- Right Gage Adze Depth Inches: depth of adzing on right gage side
- Right Field Adze Depth Inches: depth of adzing on right field side
- Tie Score: it is an integer that represents the Aurora surface score of ties with a scale of 1 to 4, 1 being the best and 4 being the worst. (-1) is for the ungraded ties
- TQI Weighted Tie Score: it is a decimal score that represents the surface score of ties with a scale of 1 to 4, 1 being the best and 4 being the worst. (-1) is for the ungraded ties
- Ballast Coverage: represents the percent of tie that is covered by ballast
- GPS QoS: quality of GPS signal, scale 1-5, 5 is best, 1 is worst
- Curvature (deg): curvature of track in degrees
- Ungraded Reason: the reason why a tie was not graded
    - 0: no obstruction
    - 1: diamond crossing
    - 2: grease mat
    - 3: plates on ties
    - 4: ballast covered
    - 5: mud
    - 6: vegetation
    - 7: other
    - 9: slab track
- Internal Tie Score: it is a decimal score that represents the x-ray score of a tie with a scale of 1 to 4, 1 being the best and 4 being the worst. (-1) is for the ungraded ties

- Aurora Xi Score: it represents the combination of surface and x-ray score (decimal), scale 1-4, 1 is best, 4 is worst, (-1) ungraded tie
- Tie State: value assigned to each tie when subdivisions of files have been merged together:
    - 0: tie is repeated in a different file
    - 1: tie is unique throughout collections
    - -1: tie was not collected, and a place marker was added after collections
    - -9: beginning or ending of section

The overall condition of the tie was given by the Tie Scores as noted above. Both integer and decimal values were assigned to each tie.

2.1 Data Preprocessing

The data was preprocessed as follows:

First, the ties were sorted in ascending order according to the milepost. Afterwards, as a part of the cleaning data process, any duplicated ties were removed and summarized with the final number of ties shown in Table 2. The dataset contained some missing values in the form of -1 for tie scores or -99 for other variables. Ties with such missing values were not deleted in order to be able to perform an accurate tie alignment.

Table 2: Summary of the number of ties 2016-2019

Year	**2016**	**2017**	**2019**
Number of ties	209175	209152	209177

The following is a year-by-year summary of the tie data.

2.1.1 Year 2016

To have a general idea of the descriptive variables in 2016, a statistical summary is presented in Figure 4. It is performed using R and it shows: the mean, standard deviation (sd), median, trimmed mean, median absolute deviation (mad), minimum value, maximum value, range, and skewness for each variable.

The Tie Number, Tie Start Slice, Tie End Slice and Cumulative distance were not included as they reset for each file.

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew
Mile.Post	1	209175	238.43	18.78	238.44	238.43	24.14	205.79	271.00	65.22	0.00
Crossing	2	209175	0.01	0.17	0.00	0.00	0.00	0.00	2.00	2.00	11.48
Switch	3	209175	0.01	0.15	0.00	0.00	0.00	0.00	2.00	2.00	12.90
Bridge	4	209175	0.01	0.16	0.00	0.00	0.00	0.00	2.00	2.00	12.70
Guard.Rail	5	209175	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	NaN
Is.Concrete	6	209175	0.00	0.02	0.00	0.00	0.00	0.00	1.00	1.00	58.06
Tie.to.Tie.Distance	7	209175	20.02	2.51	19.62	19.81	2.12	0.00	36.12	36.12	1.01
Plate.Cut.LL.Max..in.	8	209175	-15.46	35.92	-0.08	-6.97	0.12	-99.00	1.68	100.68	-1.90
Plate.Cut.LR.Max..in.	9	209175	-16.07	36.55	0.00	-7.75	0.12	-99.00	1.60	100.60	-1.83
Plate.Cut.RL.Max..in.	10	209175	-12.99	33.47	0.00	-3.91	0.12	-99.00	1.68	100.68	-2.18
Plate.Cut.RR.Max..in.	11	209175	-12.14	32.44	-0.08	-2.83	0.12	-99.00	1.76	100.76	-2.30
LRail.Diff.Plate.Cut	12	209175	-4.26	20.42	0.08	0.12	0.12	-99.00	1.52	100.52	-4.42
RRail.Diff.Plate.Cut	13	209175	-7.02	25.63	0.08	0.10	0.12	-99.00	1.76	100.76	-3.31
Joint.Bar.Tie	14	209175	0.00	0.05	0.00	0.00	0.00	0.00	1.00	1.00	18.41
Left.Plate.Type	15	209175	1.17	0.74	1.00	1.00	0.00	0.00	5.00	5.00	3.91
Right.Plate.Type	16	209175	1.15	0.73	1.00	1.00	0.00	0.00	5.00	5.00	4.48
WidthLeftSide	17	209175	99.38	7.08	100.00	99.83	5.93	54.00	230.00	176.00	-0.05
WidthRightSide	18	209175	95.54	7.21	96.00	95.93	5.93	45.00	230.00	185.00	0.09
TieLength.Feet	19	209175	8.12	0.98	8.20	8.25	0.15	-1.00	8.70	9.70	-8.35
SkewAngle.Degrees	20	209175	-1.42	9.93	-0.24	-0.35	0.58	-99.00	10.58	109.58	-9.60
Left.Field.Adze.Depth.Inches	21	209175	-0.06	0.26	0.00	0.00	0.00	-1.00	2.00	3.00	-2.95
Left.Gage.Adze.Depth.Inches	22	209175	0.00	0.15	0.00	0.00	0.00	-1.00	2.00	3.00	-3.97
Right.Gage.Adze.Depth.Inches	23	209175	-0.02	0.19	0.00	0.00	0.00	-1.00	2.00	3.00	-3.45
Right.Field.Adze.Depth.Inches	24	209175	-0.04	0.24	0.00	0.00	0.00	-1.00	2.00	3.00	-3.32
Tie.Score	25	209175	1.39	0.72	1.00	1.32	0.00	-1.00	4.00	5.00	0.27
TQI.Weighted.Tie.Score	26	209175	1.45	0.66	1.30	1.40	0.44	-1.00	4.00	5.00	-0.04
Ballast.Coverage	27	209175	4.76	9.00	1.25	2.51	1.62	-1.00	100.00	101.00	3.66
GPS.QoS	28	209175	3.65	1.13	3.00	3.68	1.48	2.00	5.00	3.00	0.16
Curvature..deg.	29	209175	-0.01	0.60	0.00	0.00	0.04	-4.67	4.70	9.37	-0.86
Ungraded.Reason	30	209175	0.01	0.17	0.00	0.00	0.00	0.00	5.00	5.00	24.08
Internal.Tie.Score	31	209175	1.31	0.64	1.10	1.21	0.15	0.00	4.00	4.00	1.80
Aurora.Xi.Score	32	209175	1.54	0.74	1.30	1.46	0.30	-1.00	4.00	5.00	0.43
Tie.State	33	209175	1.00	0.02	1.00	1.00	0.00	-9.00	1.00	10.00	-457.35

Figure 4: Summary of ties data in 2016

Table 3 shows the number of ties that are one tie from a crossing, a switch, or a bridge, (1) as well as the number of ties that are part of a crossing, a switch, or a bridge (2) in 2016. The number of ties supporting a guard rail, under a joint bar, or that are concrete is also shown below:

Table 3: Number of ties that are part of (or one tie from) a crossing, a switch, or a bridge in 2016

	Crossing	**Switch**	**Bridge**	**Guard Rail**	**Is Concrete**	**Joint Bar Tie**
1	162	37	16	0	62	612
2	1469	1217	1265	-	-	-

Table 4 describes the plate types for ties inspected in 2016 (left and right). 0 represents an unknown plate type while the other numbers represent different types as follow:

1: spike plate
2: Pandrol e-clip,
3: Pandrol fast clip
4: spike plate 18 in
5: spike plate 10 in
6: Victor plate

Table 4: Plate types for ties inspected in 2016

	0	**1**	**2**	**3**	**4**	**5**	**6**
Left Plate type	2585	191529	4855	0	7083	3123	0
Right Plate type	2595	193747	5013	0	2478	5342	0

2.1.2 Year 2017

To have a general idea of the descriptive variables in 2017, a statistics summary is presented in Figure 5. It is performed using R and it shows: the mean, standard deviation (sd), median, trimmed mean, median absolute deviation (mad), minimum value, maximum value, range, and skewness for each variable.

The Tie Number, Tie Start Slice, Tie End Slice and Cumulative distance were not included as they reset for each file.

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew
Mile.Post	1	209152	238.40	18.79	238.42	238.41	24.15	205.76	271.00	65.24	0.00
Crossing	2	209152	0.01	0.17	0.00	0.00	0.00	0.00	2.00	2.00	11.44
Switch	3	209152	0.01	0.15	0.00	0.00	0.00	0.00	2.00	2.00	12.78
Bridge	4	209152	0.01	0.12	0.00	0.00	0.00	0.00	2.00	2.00	16.30
Guard.Rail	5	209152	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	NaN
Is.Concrete	6	209152	0.00	0.02	0.00	0.00	0.00	0.00	1.00	1.00	60.03
Tie.to.Tie.Distance	7	209152	19.84	2.51	19.49	19.62	2.11	0.00	34.87	34.87	1.05
Plate.Cut.LL.Max..in.	8	209152	-6.34	24.34	0.00	0.02	0.13	-99.00	2.01	101.01	-3.54
Plate.Cut.LR.Max..in.	9	209152	-6.48	24.67	0.09	0.06	0.13	-99.00	2.28	101.28	-3.48
Plate.Cut.RL.Max..in.	10	209152	-6.88	25.38	0.09	0.07	0.13	-99.00	2.10	101.10	-3.35
Plate.Cut.RR.Max..in.	11	209152	-7.55	26.33	0.00	0.00	0.13	-99.00	2.45	101.45	-3.19
LRail.Diff.Plate.Cut	12	209152	-3.02	17.45	0.09	0.14	0.13	-99.00	1.84	100.84	-5.32
RRail.Diff.Plate.Cut	13	209152	-5.53	23.06	0.09	0.14	0.13	-99.00	2.36	101.36	-3.81
Joint.Bar.Tie	14	209152	0.00	0.07	0.00	0.00	0.00	-1.00	1.00	2.00	13.09
Left.Plate.Type	15	209152	1.11	0.61	1.00	1.00	0.00	-1.00	6.00	7.00	4.97
Right.Plate.Type	16	209152	1.13	0.65	1.00	1.00	0.00	-1.00	6.00	7.00	4.60
WidthLeftSide	17	209152	110.05	7.76	111.00	110.56	5.93	-1.00	256.00	257.00	-0.89
WidthRightSide	18	209152	110.04	7.77	111.00	110.55	5.93	-1.00	256.00	257.00	-0.88
TieLength.Feet	19	209152	8.42	1.01	8.40	8.52	0.15	-1.00	9.40	10.40	-8.34
SkewAngle.Degrees	20	209152	-1.75	9.84	-0.57	-0.70	0.61	-99.00	9.46	108.45	-9.64
Left.Field.Adze.Depth.Inches	21	209152	-0.01	0.16	0.00	0.00	0.00	-1.00	2.00	3.00	-4.31
Left.Gage.Adze.Depth.Inches	22	209152	-0.01	0.17	0.00	0.00	0.00	-1.00	2.00	3.00	-2.89
Right.Gage.Adze.Depth.Inches	23	209152	-0.01	0.18	0.00	0.00	0.00	-1.00	2.00	3.00	-1.98
Right.Field.Adze.Depth.Inches	24	209152	-0.01	0.15	0.00	0.00	0.00	-1.00	2.00	3.00	-4.22
Tie.Score	25	209152	1.39	0.74	1.00	1.30	0.00	-1.00	4.00	5.00	0.63
TQI.Weighted.Tie.Score	26	209152	1.42	0.70	1.10	1.34	0.15	-1.00	4.00	5.00	0.45
Ballast.Coverage	27	209152	1.86	5.43	0.54	0.82	0.68	-1.00	100.00	101.00	9.44
GPS.QoS	28	209152	4.11	1.37	5.00	4.26	0.00	-1.00	5.00	6.00	-0.89
Curvature..deg.	29	209152	-0.01	0.61	0.00	0.00	0.06	-4.64	4.98	9.62	-0.89
Ungraded.Reason	30	209152	0.00	0.12	0.00	0.00	0.00	-1.00	5.00	6.00	35.43
Internal.Tie.Score	31	209149	1.60	0.82	1.30	1.48	0.30	-1.00	4.00	5.00	0.91
Aurora.Xi.Score	32	209149	1.76	0.85	1.50	1.66	0.59	-1.00	4.00	5.00	0.54
Tie.State	33	209152	1.00	0.04	1.00	1.00	0.00	-9.00	1.00	10.00	-115.04

Figure 5: Summary of ties data in 2017

Table 5 shows the number of ties that are one tie from a crossing, a switch, or a bridge, (1) as well as the number of ties that are part of a crossing, a switch, or a bridge (2) in 2017. The number of ties supporting a guard rail, under a joint bar, or that are concrete is also shown in the table.

Table 5: Number of ties that are part of (or one tie from) a crossing, a switch, or a bridge in 2017

	Crossing	Switch	Bridge	Guard Rail	Is Concrete	Joint Bar Tie
1	164	35	10	0	58	880
2	1480	1240	773	-	-	-

Table 6 describes the plate types for ties inspected in 2017 (left and right). 0 represents an unknown plate type while the other numbers represent different types as follow:

1: spike plate,
2: Pandrol e-clip,
3: Pandrol fast clip,
4: spike plate 18 in,
5: spike plate 10 in
6: Victor plate

Table 6: Plate types for ties inspected in 2017

	0	1	2	3	4	5	6
Left plate type	2594	196356	3906	0	4020	2161	60
Right Plate type	2529	194235	5141	1	4589	2534	68

2.1. 3 Year 2019

To have a general idea of the descriptive variables in 2019, a statistics summary is presented in Figure 6. It is performed using R and it shows: the mean, standard deviation (sd), median, trimmed mean, median absolute deviation (mad), minimum value, maximum value, range, and skewness for each variable.

The Tie Number, Tie Start Slice, Tie End Slice and Cumulative distance were not included as they reset for each file.

	vars	n	mean	sd	median	trimmed	mad	min	max	range	skew
Mile.Post	1	209177	238.50	18.82	238.51	238.50	24.22	205.94	271.07	65.13	0.00
Crossing	2	209177	0.01	0.17	0.00	0.00	0.00	0.00	2.00	2.00	11.62
Switch	3	209177	0.01	0.15	0.00	0.00	0.00	0.00	2.00	2.00	13.12
Bridge	4	209177	0.01	0.11	0.00	0.00	0.00	0.00	2.00	2.00	17.57
Guard.Rail	5	209177	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	NaN
Is.Concrete	6	209177	0.00	0.02	0.00	0.00	0.00	0.00	1.00	1.00	52.78
Tie.to.Tie.Distance	7	209177	19.72	2.42	19.39	19.50	2.00	0.00	34.11	34.11	1.16
Plate.Cut.LL.Max..in.	8	209177	-23.85	42.36	-0.08	-17.48	0.23	-99.00	1.89	100.89	-1.21
Plate.Cut.LR.Max..in.	9	209177	-10.27	30.28	0.00	-0.52	0.12	-99.00	1.89	100.89	-2.59
Plate.Cut.RL.Max..in.	10	209177	-10.04	30.00	0.00	-0.23	0.12	-99.00	1.82	100.82	-2.63
Plate.Cut.RR.Max..in.	11	209177	-19.84	39.63	-0.08	-12.46	0.12	-99.00	1.82	100.82	-1.50
LRail.Diff.Plate.Cut	12	209177	-8.00	27.24	0.08	0.12	0.12	-99.00	1.82	100.82	-3.04
RRail.Diff.Plate.Cut	13	209177	-8.67	28.23	0.08	0.12	0.12	-99.00	1.97	100.97	-2.89
Joint.Bar.Tie	14	209177	0.00	0.06	0.00	0.00	0.00	0.00	1.00	1.00	17.06
Left.Plate.Type	15	209177	1.24	0.91	1.00	1.00	0.00	0.00	6.00	6.00	3.27
Right.Plate.Type	16	209177	1.26	0.95	1.00	1.00	0.00	0.00	6.00	6.00	3.20
WidthLeftSide	17	209177	112.56	9.30	114.00	113.12	7.41	86.00	263.00	177.00	1.17
WidthRightSide	18	209177	112.57	9.30	114.00	113.13	7.41	86.00	263.00	177.00	1.16
TieLength.Feet	19	209177	8.23	1.34	8.40	8.45	0.15	-1.00	9.40	10.40	-5.81
SkewAngle.Degrees	20	209177	-2.48	12.62	-0.79	-0.84	1.10	-99.00	13.92	112.92	-7.45
Left.Field.Adze.Depth.Inches	21	209177	-0.05	0.26	0.00	0.00	0.00	-1.00	2.00	3.00	-2.83
Left.Gage.Adze.Depth.Inches	22	209177	-0.01	0.20	0.00	0.00	0.00	-1.00	2.00	3.00	-2.99
Right.Gage.Adze.Depth.Inches	23	209177	-0.01	0.19	0.00	0.00	0.00	-1.00	2.00	3.00	-3.32
Right.Field.Adze.Depth.Inches	24	209177	-0.02	0.21	0.00	0.00	0.00	-1.00	2.00	3.00	-3.23
Tie.Score	25	209177	1.51	0.81	1.00	1.44	0.00	-1.00	4.00	5.00	0.04
TQI.Weighted.Tie.Score	26	209177	1.53	0.75	1.40	1.49	0.59	-1.00	4.00	5.00	-0.18
TQI.Plate.Cut.Tie.Score	27	209177	1.58	0.75	1.40	1.54	0.59	-1.00	4.00	5.00	-0.29
Ballast.Coverage	28	209177	5.30	8.75	1.78	3.31	2.46	-1.00	95.97	96.97	3.28
GPS.QoS	29	209177	3.00	0.00	3.00	3.00	0.00	2.00	3.00	1.00	-323.39
Curvature..deg.	30	209177	-0.01	0.61	0.00	0.00	0.04	-4.85	4.73	9.58	-0.73
Ungraded.Reason	31	209177	0.02	0.27	0.00	0.00	0.00	0.00	7.00	7.00	17.00
Internal.Tie.Score	32	209177	1.36	0.73	1.20	1.27	0.30	-1.00	4.00	5.00	0.78
Aurora.Xi.Score	33	209177	1.71	0.82	1.60	1.66	0.59	-1.00	4.00	5.00	0.00
Tie.State	34	209177	1.00	0.03	1.00	1.00	0.00	-9.00	1.00	10.00	-323.39

Figure 6: Summary of ties data in 2019

Table 7 shows the number of ties that are one tie from a crossing, a switch, or a bridge, (1) as well as the number of ties that are part of a crossing, a switch, or a bridge (2) in 2019. The number of ties supporting a guard rail, under a joint bar, or that are concrete is also shown in Table 7.

Table 7: Number of ties that are part of (or one tie from) a crossing, a switch, or a bridge in 2019

	Crossing	**Switch**	**Bridge**	**Guard Rail**	**Is Concrete**	**Joint Bar Tie**
1	159	33	10	0	75	711
2	0	1177	666	-	-	-

Table 8 describes the plate types for ties inspected in 2019 (left and right). 0 represents an unknown plate type while the other numbers represent different types as follow:

1: spike plate
2: Pandrol e-clip,
3: Pandrol fast clip
4: spike plate 18 in
5: spike plate 10 in
6: Victor plate

Table 8: Plate types for ties inspected in 2019

	0	**1**	**2**	**3**	**4**	**5**	**6**

left plate type	4260	186590	4074	0	7204	6942	106
Right Plate type	3865	185279	5318	0	6356	8260	98

3. Data Processing

Once the tie data was properly cleaned and identified, the files were divided into individual miles, as defined by the data ID. Thus, each mile had a variable number of ties, depending on the defined start-stop location.

The mile-by-mile summary, to include number of ties in each tie condition category (1 through 4) is presented in Table 9 below. A summary of the tie data is presented in Table 10.

The data set represents 67 miles of tie condition data, mileposts started at 205 and ended at 272.

Table 9: Mile by mile summary

		2016						2017						2019					
Milepost		Number of Ties	Tie Score					Number of Ties	Tie Score					Number of Ties	Tie Score				
			-1	1	2	3	4		-1	1	2	3	4		-1	1	2	3	4
205	206	768	66	592	92	15	3	857	106	635	92	18	6	198	32	138	23	4	1
206	207	2807	7	1793	848	149	10	2808	3	1712	846	211	36	3178	161	1759	953	253	52
207	208	3433	43	2193	1012	163	22	3433	45	1926	1121	289	52	3230	31	1486	1333	337	43
208	209	3037	11	2030	876	111	9	3037	12	1761	1006	242	16	3248	40	1397	1425	367	19
209	210	2998	14	2167	730	79	8	2998	23	2071	732	162	10	3136	21	2750	319	38	8
210	211	3275	15	2267	886	97	10	3275	60	2125	897	173	20	3259	60	2854	287	55	3
211	212	3379	23	2214	928	198	16	3368	22	2081	821	401	43	3286	31	1492	1184	512	67
212	213	3149	22	2146	801	160	20	3157	14	1977	854	278	34	3267	85	1474	1261	402	45
213	214	3196	39	2362	711	77	7	3194	47	2193	754	180	20	3243	44	1773	1139	255	32
214	215	3184	59	2077	858	166	24	3185	62	2084	646	343	50	3240	103	1765	920	408	44
215	216	3278	29	2205	890	133	21	3277	29	2110	908	197	33	3230	43	1553	1335	281	18
216	217	2612	23	1603	766	195	25	2610	12	1650	757	167	24	3249	37	1965	1031	197	19
217	218	3786	122	2852	689	111	12	3778	131	2987	577	68	15	3231	145	2487	532	56	11
218	219	3380	14	2708	549	96	13	3381	15	2696	549	102	19	3206	34	2328	713	116	15
219	220	3243	79	2368	653	128	15	3243	53	2560	512	98	20	3211	126	2282	649	143	11
220	221	3118	16	2375	583	121	23	3122	18	2310	632	119	43	3196	32	2212	733	182	37
221	222	3246	15	2112	955	138	26	3248	19	1952	1065	175	37	3246	31	2072	957	159	27
222	223	3263	138	2061	875	163	26	3262	109	2044	875	195	39	3223	160	2468	508	79	8
223	224	3097	28	1867	987	196	19	3103	35	1768	1043	217	40	3182	54	1623	1188	294	23
224	225	3206	83	2537	468	101	17	3203	82	2597	404	92	28	3157	90	2297	626	120	24
225	226	3184	65	2644	305	140	30	3184	20	2644	339	133	48	3107	28	2363	499	157	60
226	227	2961	4	2493	311	113	40	2964	4	2466	336	112	46	3105	8	2274	624	144	55
227	228	3294	3	2361	757	158	15	3292	2	2414	661	189	26	3168	8	1831	1056	250	23
228	229	3364	60	1563	1373	336	32	3330	29	1712	1135	384	70	3247	35	1305	1417	455	35
229	230	3126	3	937	1757	394	35	3133	2	1627	1055	393	56	3227	14	1254	1436	489	34
230	231	3271	196	487	1810	740	38	3271	1	1454	1090	566	160	3248	5	1039	1404	722	78
231	232	3222	52	1151	1576	415	28	3221	65	1722	1030	348	56	3215	55	1075	1552	493	40
232	233	3244	7	1246	1469	481	41	3246	5	1780	996	365	100	3248	12	829	1744	590	73
233	234	3579	110	1354	1764	339	12	3572	92	2615	715	134	16	3247	107	1629	1231	263	17
234	235	3156	93	701	1765	586	11	3154	79	2009	774	242	50	3179	94	1744	974	338	29
235	236	2993	33	1522	1060	338	40	2993	34	1853	730	282	94	3182	57	1583	1080	409	53
236	237	3230	133	1338	1368	372	19	3228	120	1954	862	254	38	3174	146	1637	1020	330	41
237	238	3099	24	1745	991	308	31	3102	19	1751	877	364	91	3178	26	1418	1195	465	74
238	239	3182	132	1627	1048	325	50	3178	47	1761	949	325	96	3194	93	1451	1095	473	82
239	240	3220	443	1919	684	147	27	3214	489	2080	505	109	31	3156	494	1910	610	107	35
240	241	3150	233	1741	921	223	32	3157	205	2457	359	107	29	3183	287	2278	464	116	38
241	242	3288	150	1928	989	204	17	3286	176	2638	380	73	19	3210	171	2155	744	125	15
242	243	2927	119	1671	860	258	19	3008	137	2298	437	123	13	3225	118	2130	768	182	27
243	244	3313	16	2568	657	70	2	3310	21	2740	481	66	2	3218	32	2194	904	88	0
244	245	3283	19	2088	1010	160	6	3278	18	1861	1168	204	27	3211	37	1870	1054	223	27
245	246	3204	14	2137	937	113	3	3203	13	1959	995	221	15	3239	36	1508	1290	391	14
246	247	3400	30	1887	1316	162	5	3402	25	1728	1318	309	22	3247	33	1000	1709	475	30
247	248	3114	15	1737	1137	216	9	3111	16	1541	1224	293	37	3236	48	883	1674	585	46
248	249	3502	30	2350	920	189	13	3506	27	2325	894	230	30	3232	224	1451	1288	258	11
249	250	2905	37	1687	1068	111	2	2897	28	1541	1098	219	11	3203	75	1051	1656	406	15
250	251	3268	32	1569	1302	348	17	3269	16	1431	1393	390	39	3185	50	1084	1565	461	25
251	252	3210	13	1614	1166	388	29	3208	1	1559	1160	421	67	3222	29	1037	1496	596	64
252	253	3367	178	2232	679	261	17	3275	43	2362	696	156	18	3223	15	2276	690	221	21
253	254	3362	14	2361	728	244	15	3361	29	2210	846	240	36	3210	36	1735	1089	314	36
254	255	2956	26	1832	750	331	17	2954	24	1576	890	391	73	3190	40	1178	1303	587	82
255	256	3241	73	1796	1037	302	33	3240	103	1667	1011	372	87	3172	109	1031	1355	566	111

256	257	3183	16	1931	1080	154	2	3186	20	1842	1017	276	31	3190	67	1166	1462	453	42
257	258	3264	20	2062	979	198	5	3258	22	2018	936	259	23	3181	47	1235	1459	399	41
258	259	3289	330	1848	860	244	7	3296	255	1787	961	266	27	3240	319	1599	995	307	20
259	260	3078	105	1553	1072	333	15	3079	92	1541	1063	351	32	3200	131	1065	1384	559	61
260	261	3326	69	2255	905	96	1	3327	1	2197	1012	111	6	3306	191	1379	1460	270	6
261	262	3073	21	2246	697	107	2	3074	14	2287	646	120	7	3127	41	1796	1089	193	8
262	263	3211	26	2385	720	78	2	3211	24	2413	669	94	11	3175	33	1803	1157	171	11
263	264	3302	43	2558	621	77	3	3292	22	2395	774	94	7	3321	65	1762	1278	201	15
264	265	3196	31	2391	719	51	4	3194	36	2368	691	92	7	3205	51	1740	1213	185	16
265	266	3447	26	2143	1151	121	6	3444	30	2000	1170	222	22	3270	39	1230	1550	417	34
266	267	3352	50	2130	1077	92	3	3358	47	1990	1042	261	18	3294	52	1240	1479	482	41
267	268	3095	27	2216	741	100	11	3088	28	2236	693	126	5	3210	71	1487	1369	264	19
268	269	3164	115	2377	629	43	0	3161	118	2299	650	90	4	3209	127	1702	1196	174	10
269	270	3153	3	2599	534	17	0	3154	3	2578	514	56	3	3157	6	2103	950	97	1
270	271	2957	229	2328	337	53	10	2944	218	2210	435	65	16	3180	309	2288	531	44	8
271	272	15	0	8	6	1	0	0	0	0	0	0	0	215	131	73	10	1	0

Table 10 below shows the total number of ties and their condition score for each year:

Table 10: total number of ties and their condition score for each year

2016						2017						2019					
Total Number of Ties	Tie Score					Total Number of Ties	Tie Score					Total Number of Ties	Tie Score				
	-1	1	2	3	4		-1	1	2	3	4		-1	1	2	3	4
209175	4314	129845	60800	13134	1082	209152	3717	135135	53768	14225	2307	209177	5562	110046	71684	19754	2131

Figure 7 below summarizes the ties with scores 1, 2, 3 or 4 for the 3 years: 2016, 2017, 2019

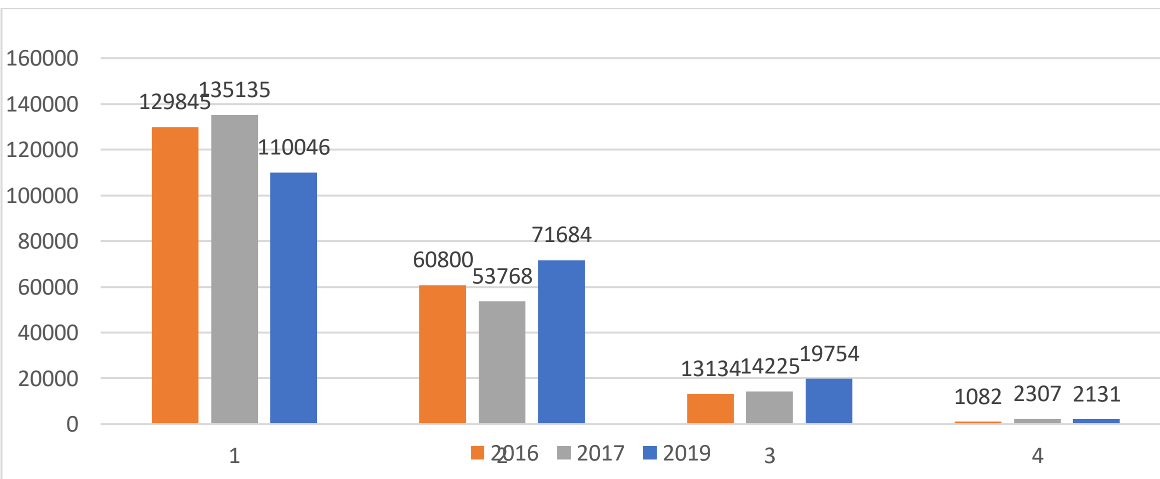


Figure 7: ties with scores 1, 2, 3 or 4 for years: 2016, 2017, 2019

3.1 Average Tie Score Per Mile

To better understand the behavior of the track's ties on an "overview" or macro level basis, the average tie condition per mile was computed in order to evaluate the rate of tie degradation. The arithmetic average for each mile was calculated using the following equation:

$$Avg = \frac{\sum_{i=1}^{i=n} Xi}{n}$$

where:

X_i is the tie score for tie-i
n is the total number of ties per mile.

Note: ties with a score of -1 were not considered as they represented a tie where Aurora was unable to get a good condition value.

In order to evaluate the effect of tie conditions, several different sets of analyses were performed comparing average tie condition (per mile) with various other parameters.

Figure 8 presents the average tie condition on a mile-by-mile basis for each year data is available: The degradation with time is clearly evident. In addition, those miles where significant tie replacement, such as with a tie gang, has been performed are also evident by the fact that the average tie condition improves with time, corresponding to the introduction of new replacement ties in that mile. Thus, it appears that a tie gang was run between MP 5 and 7 in 2018, and another tie gang run between MP 35 and 39 between 2016 and 2017. Other MP suggest some spot tie replacement to eliminate failed ties (“4s”) as well.

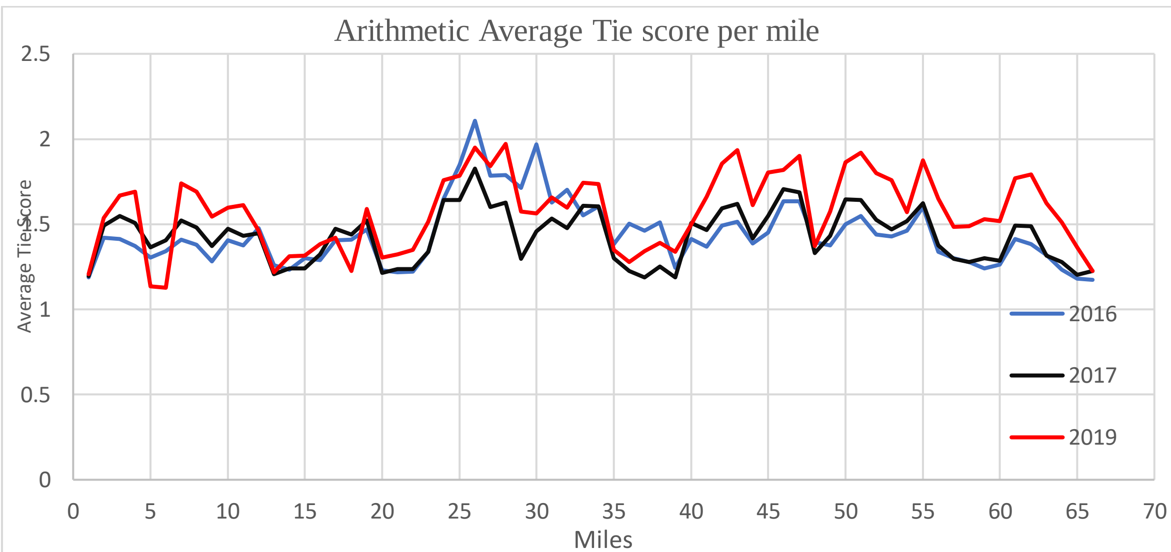


Figure 8: Average tie condition on a mile by mile basis for each year

From Figure 8, it is possible to classify the individual miles into several different categories based on suspected tie replacement activity, as follows:

- The miles where no tie gang appears to have worked are the miles with a clear tie score average increase as observed throughout the three years:
    - 1 : Average tie score 2019> Average tie score 2017 > Average tie score 2016

- The miles where a tie gang appears to have worked between 2016 and 2017 (defined in sub bullet 3 below) or where there was a significant number of spot tie replacements during that period (defined in sub bullet 2 below) as suggested by a sudden improvement (decrease) of the tie score and described as follows:

- 2 : Average tie score 2019> Average tie score 2016 > Average tie score 2017
- 3 : Average tie score 2016> Average tie score 2019 > Average tie score 2017

- The miles where a tie gang happened between 2017 and 2019 (defined in sub bullet 4 below) or where there was a significant number of spot tie replacements during that period (defined in sub bullet 5 below) as suggested by a sudden improvement (decrease) of the tie score and described as follows:
  - 4 : Average tie score 2017 >Average tie score 2016 > Average tie score 2019
  - 5 : Average tie score 2017 >Average tie score 2019 > Average tie score 2016

The classification of the mile into the corresponding class (defined as 1-5 above) is important because it allows for a better understanding of the ties' behavior, and helps identify those sections of the track where no tie gang or significant tie replacement activity occurred.
The classification is summarized in Table 11:

Table 11: Summary of Miles Classification

	Classes				
Miles	**1**	**2**	**3**	**4**	**5**
1	X				
2	X				
3	X				
4	X				
5				X	
6				X	
7	X				
8	X				
9	X				
10	X				
11	X				
12			X		
13			X		
14	X				
15		X			
16	X				
17					X
18				X	
19	X				
20		X			
21	X				
22	X				
23		X			
24		X			
25			X		
26			X		
27		X			
28		X			
29			X		
30			X		
31		X			
32			X		
33	X				
34		X			
35			X		
36			X		
37			X		
38			X		
39		X			
40					X
41	X				
42	X				
43	X				
44	X				
45	X				
46	X				
47	X				
48			X		
49	X				
50	X				
51	X				
52	X				
53	X				
54	X				
55	X				

56	X				
57		X			
58	X				
59	X				
60	X				
61	X				
62	X				

63		X			
64	X				
65	X				
66					X
67			X		

The total number of miles belonging to each category is summarized in Table 12 below:

Table 12: Total number of miles in each category

Category	**1**	**2**	**3**	**4**	**5**
Total number of miles	37	11	13	3	3

As can be seen from the tables above, there are 37 miles that belong to category 1, which represents about 55 % of the studied track, while the remaining 30 miles belong to the other categories. The analysis will focus on the portion of the track that belong to category 1 where no tie gang or significant number of spot tie replacement occurred. Thus, the focus of this analysis will be on the 37 miles of track where no major tie replacement occurred.

4. Tie Alignment

In order to perform a detailed, tie-by-tie analysis, it is necessary to accurately align the ties, so that each individual tie can be followed comparatively over the three-year time period. This section presents the results of this tie alignment process.

The goal of tie alignment is to be able to identify and "follow" individual ties through the three-years of data. Although milepost information was provided, it is not accurate to the individual tie level so that more accurate alignment is necessary.

As noted, this analysis will now focus on the 37 miles (approximately 120,000 ties) for which no major tie replacement or maintenance activity has occurred during the three year study period. Those miles determined to have had major tie replacement, as discussed above, were not included in the analysis going forward. As such, alignment was restricted to these 37 miles for which minimum spot tie replacement occurred and where the tie-to-tie distance can be used to align ties. Tie condition scores where then used to confirm the alignment.

The data alignment was performed on an individual mile basis using the following steps:

- For each of the identified miles, an initial sort was performed by year and by milepost.
- Using the 2016 data, the ties in each mile were indexed from 1 to N, where N is the total number of ties in the mile
- Using a cross-correlational function[3] the shift between each mile's 2016 condition and the same mile's 2019 condition was determined. Note, the shift or lag ranged from small to

[3] Available in "R" software package

relatively large, more than 100 ties.

- Each miles' 2019 ties were then indexed using this calculated lag or shift.
- After the initial alignment, each mile was divided into 50 tie groups, with a unique numeric identifier for each group.
- Each 50 ties group was indexed from 1 to 50 and then the same cross-correlation function used to calculate the lag or shift between 2016 and 2019 data. Note, the individual 50-tie group lag was usually less than 10 ties.
- The individual tie identifier or indices for the 2019 tie data were then shifted to match the ties indices of 2016 data with the shift varying per the calculated lag.
- The alignment was checked using the Cross-Correlation Function on the tie condition values.
- A final visual check was performed. In a few cases, the location indices needed to be adjusted by +/- 1 tie. This was because the Aurora test measurements reading were performed in opposite directions in 2016 and 2019 (increasing versus decreasing milepost direction of travel). Thus, while the tie to tie distance was similar, the buildup of deviation from 19 ½" spacing resulted in the need for an occasional shift in tie ID by 1 tie.

Figures 9-14 present examples of tie alignment using the cross-correlation function.

Figure 9 shows the "before alignment" tie to tie distance (y axis) for the first 800 ties for mile 33. Note the red line is 2016 data and the black line 2019 data.

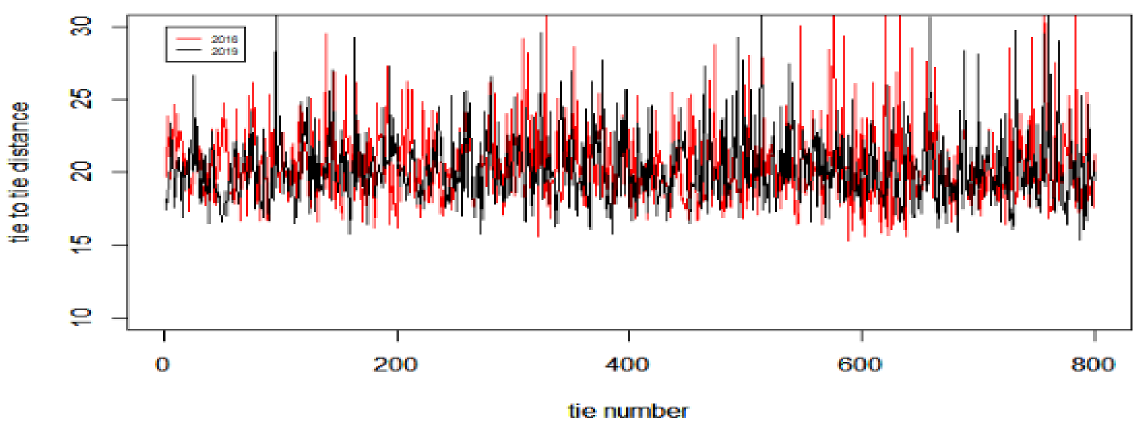


Figure 9: Tie to tie distance before alignment (800 ties)

It can be seen that the two plots in Figure 9 (tie-to-tie distance of 2016 and 2019) are not alligned. Using the cross correlation function (CCF) for the tie-to-tie distance parameter in 2016 and 2019 gives the results per lag value as shown in Figure 10.

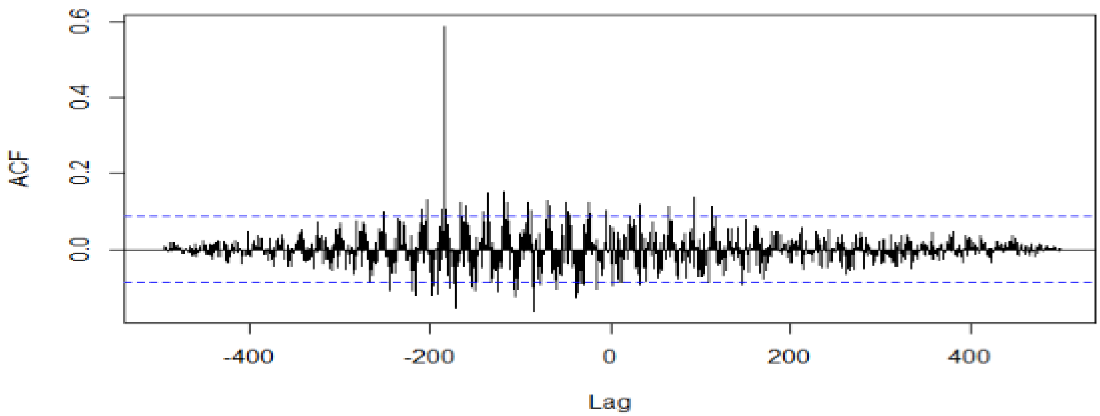


Figure 10: Cross Correlation Function result in R

The lag with the maximum ACF [4]coefficient is -185. That means that the ties'indices for 2019 have to be shifted by -185 feet to be aligned with those of 2016.

After adjusting the indices, the plots are aligned as shown in Figure 11. While the alignment appears to be much better, there is still a lag in some locations. This is due to missing individual ties or encoder slip from one inspection to another.

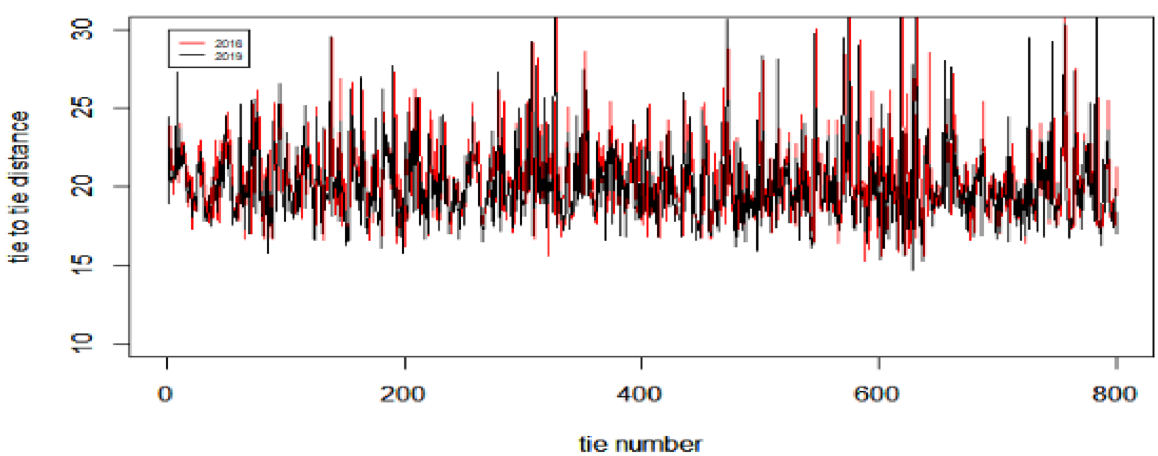


Figure 11: Tie to tie distance after alignment (800 ties)

After aligning the entire mile, and shifting the indices, a second, smaller scale alignment was performed on a 50 per 50 ties basis using the tie number and the tie to tie distance for 2016 and 2019 as shown in Figure 12.

[4] ACF stands for Autocorrelation Function

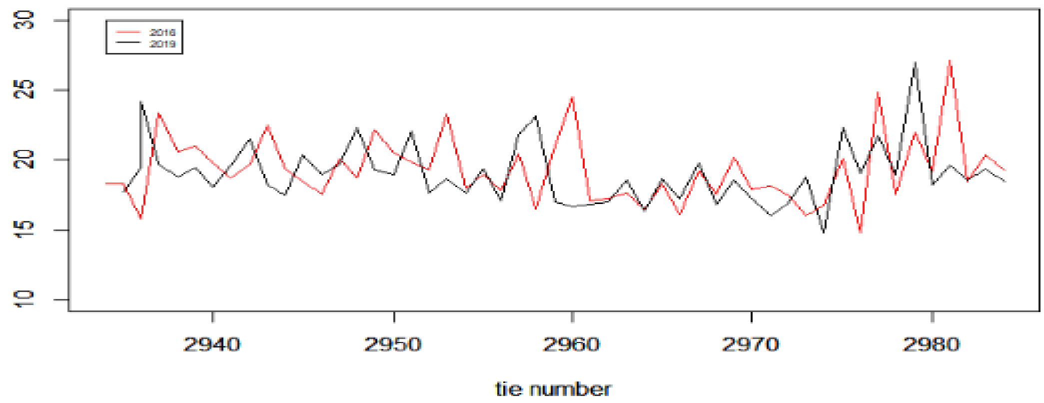


Figure 12: Tie to tie distance before alignment (50 ties)

As can be seen in Figure 12, the two plots (Tie number vs. tie-to-tie distance of 2016 and 2019) are not yet perfectly alligned.

Again using the cross correlation function (CCF) for the tie-to-tie distance parameter in 2016 and 2019 gives the following results (Figure 13):

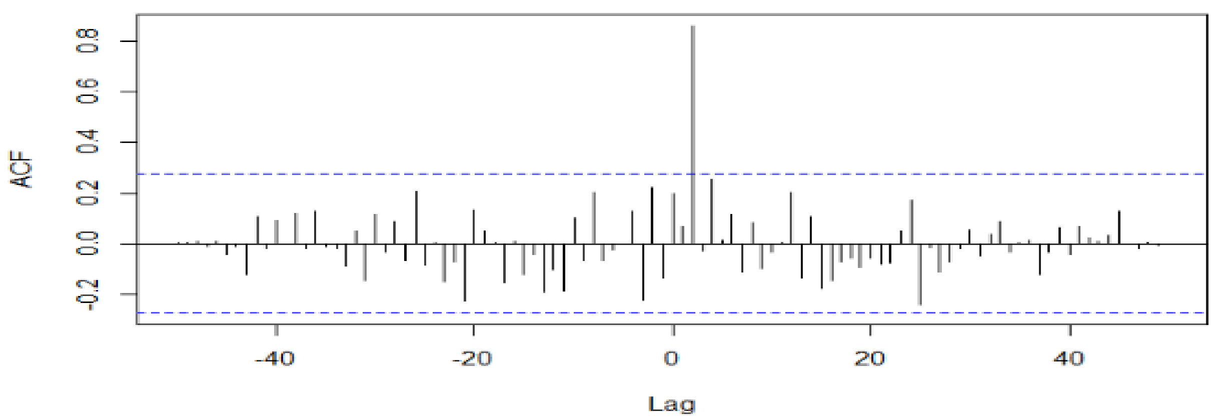


Figure 13: Cross Correlation Function result in R (for 50 ties)

The lag with the maximum ACF coefficient is +2. That means that the ties'indices of 2019 should be shifted by +2 to be aligned with those of 2016 for this particular 50 foot window.

After adjusting the tie indices for 2019, the plots are aligned as shown in Figure 14**:**

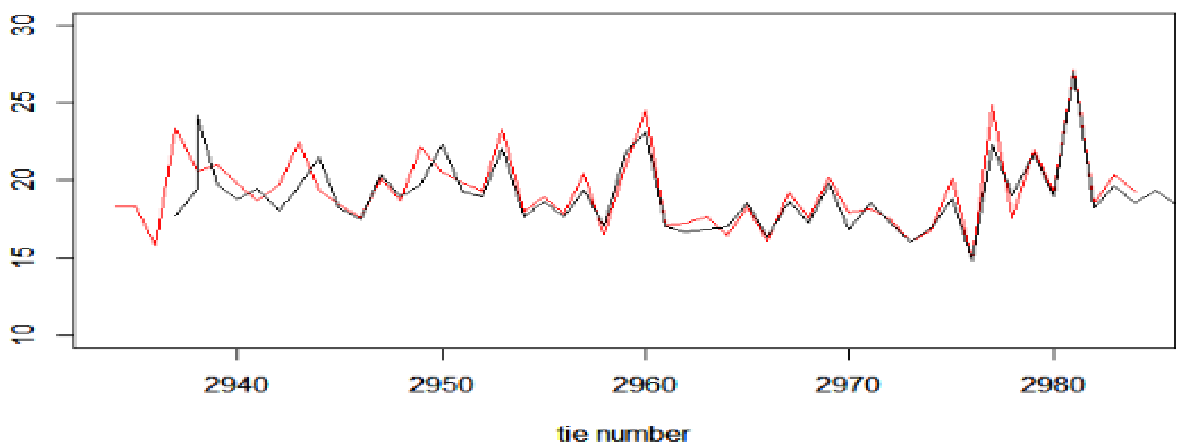


Figure 14: Tie to tie distance after alignment (50 ties)

4.1 Finalization of Tie Alignment

In order to verify the tie alignment for each 50-ties subset, a second alignment analysis was performed as illustrated in Figures 15-17. Figure 15 presets the results of the alignment performed previously, where the tie condition score (1 through 4) is plotted against the tie number for the aligned ties.

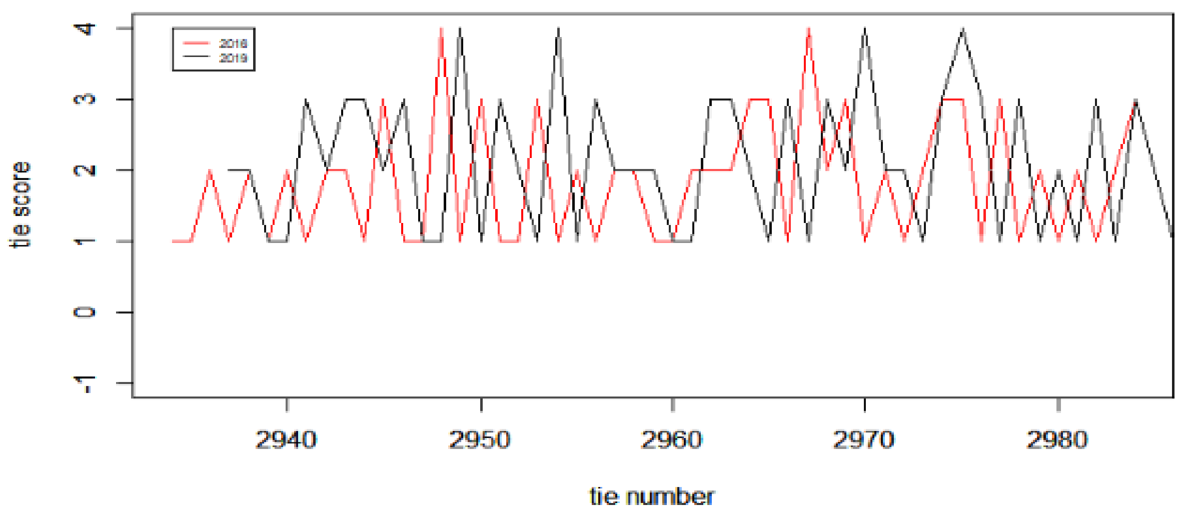


Figure 15: Tie condition score against tie number for the aligned ties

Using the cross-correlation function for the two sets of tie condition data in Figure 15 (tie condition for 2016 and 2019) gives the result presented in Figure 16.

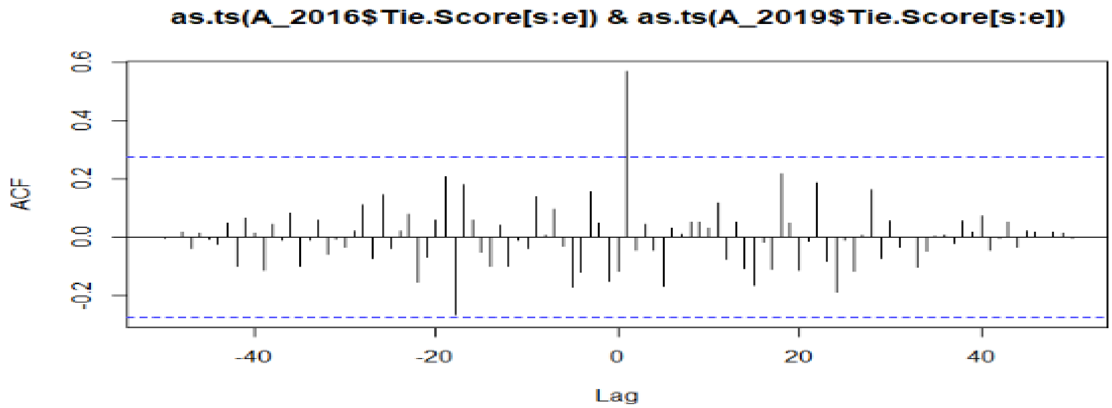


Figure 16: Cross Correlation Function result in R (for tie score)

As shown in Figure 16, the lag with the maximum ACF coefficient is +1. This means that the ties'indices of 2019 should be shifted by +1 to be aligned with those of 2016. Plotting the final tie position vs their respective tie condition scores gives the graph presented in Figure 17.

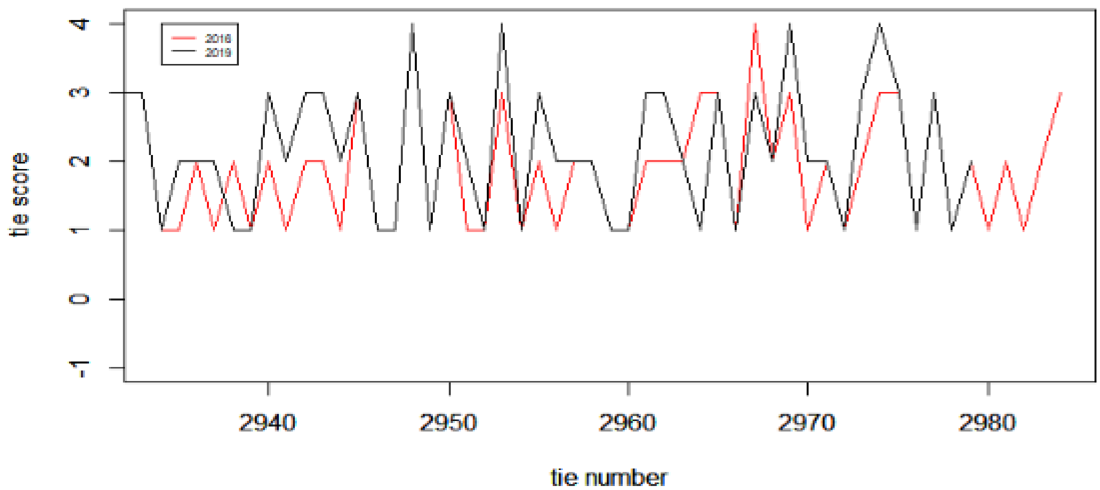


Figure 17: Final tie position vs respective tie condition scores

It is to be noted that not all the miles had an index difference of +/-1 in this last step. Some miles did not require a modification of the indices based on the tie score, since the initial alignment steps produced an accurate alignment of the two years' worth of data. The steps described above were repeated for all 37 study miles; i.e. those miles in which no tie gang worked as determined by the per mile weighted tie condition averages discussed previously.

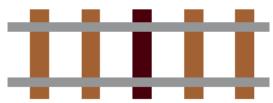
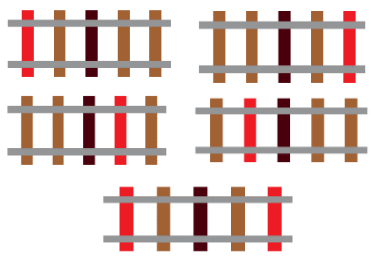
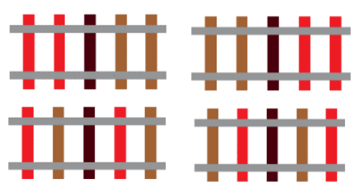
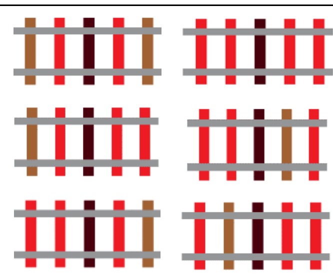
4. Cleaned Dataset

4.1. Study Groups

After cleaning the data, dismissing any individual tie for which tie replacement occurred during the three years interval, and aligning the ties, there remained 96,421 ties in the study data set. These ties were then divided into 4 different groups depending on the adjacent tie condition and the associated loss of support (percentage) that resulted.

The four study tie groups defined by average loss of support, are shown in Table 13, along with the number of ties belonging to each group.

Table 13: Average loss of support and ties configurations corresponding to each group

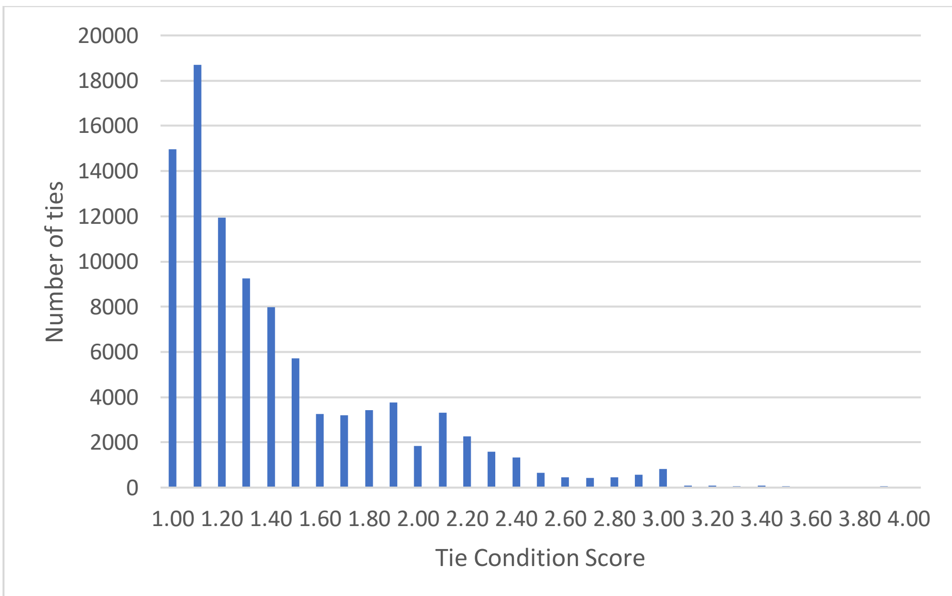
Group	Configuration	Average Loss of support (%)	Number of ties in the category
F		0	77937
A		17	16379
B		33	1410
C		46	695

Note, for Group F, all adjacent ties are in good condition, and the center tie has full support from its surrounding ties. Groups A, B, and C have increasing numbers of failed adjacent ties and corresponding increasing loss of support, as calculated from Beam On Elastic Foundation (BOEF) theory and Figure 1. Thus, the worst condition, where all four adjacent ties have failed (two on each side) is Category C, with a calculated loss of support of 46 %[5].

5.2. Tie Condition Distribution

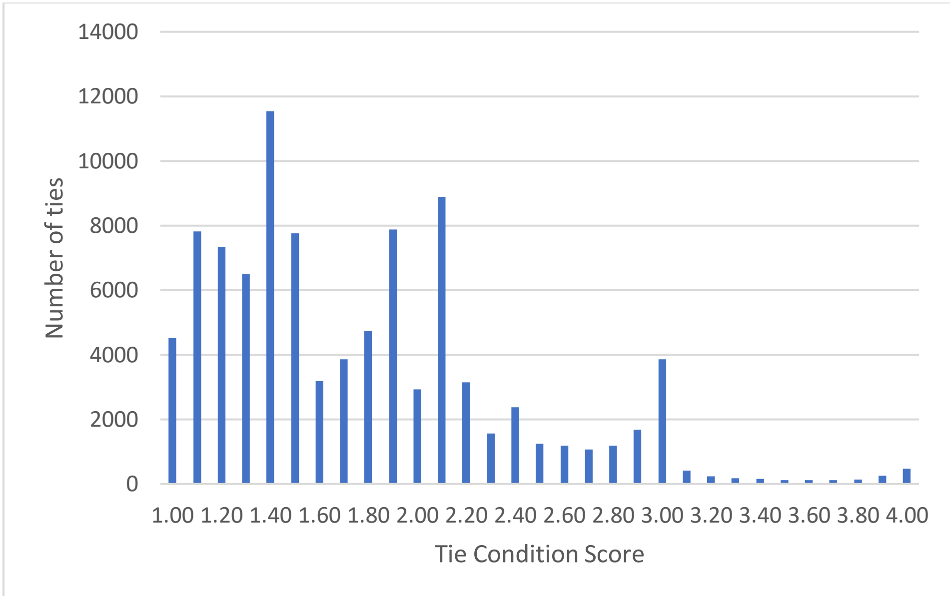
In order to analyze tie scores changes in each group between 2016 and 2019, distribution histograms were generated.

Distribution graphs for all tie condition scores at the decimal level are presented in Figures 18 A and B for both 2016 and 2019, respectively.



A: Tie score distribution in 2016

[5] Note; this percentage represents the weighted average of the loss of support for the six configurations represented by Category C. Weighted average was based on the number of ties in each of these six configurations.



B: Tie score distribution in 2019

Figure 18: Tie Condition score histogram for all groups

To better visualize the tie condition degradation over the three year period, the Tie Condition score distribution for 2016 and 2019 were plotted on the same graph and are represented in Figure 19 below.

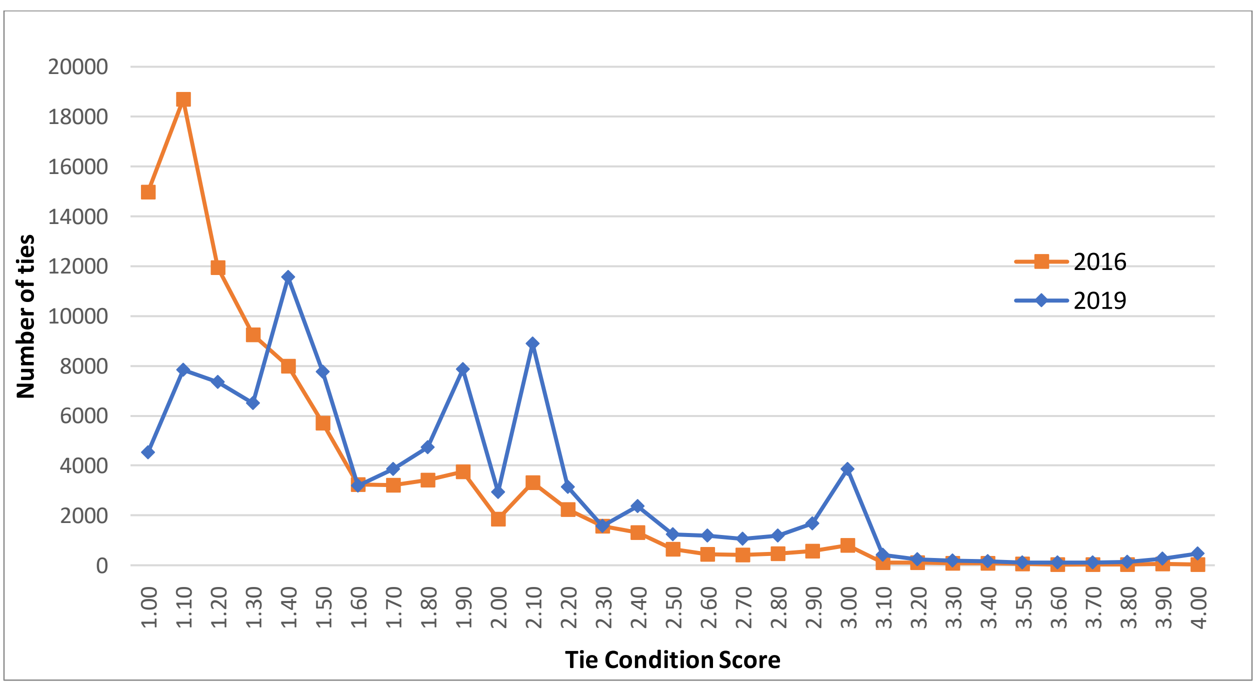


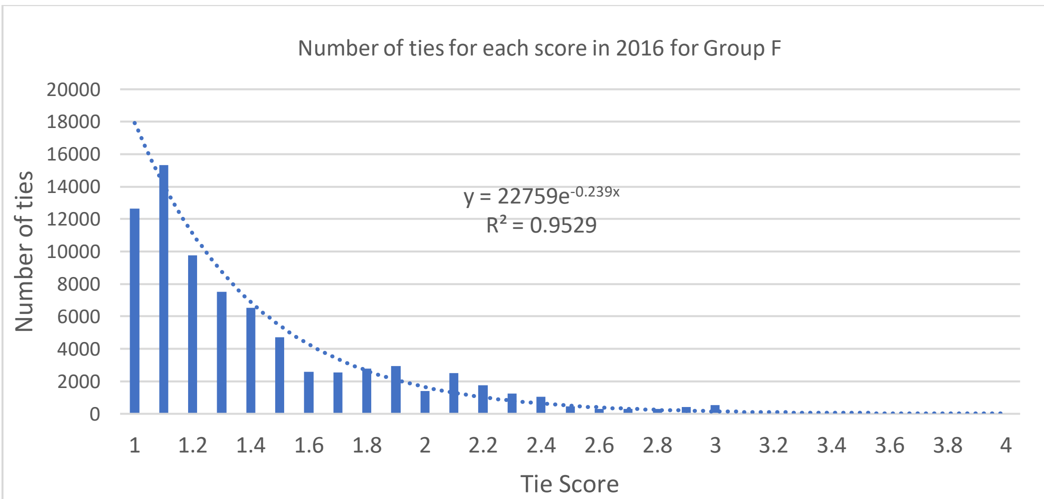
Figure 19: Tie Condition score distribution for 2016 and 2019

From Figure 19, it can be noted that there is an overall tie condition degradation as the number of "good" ties decreases from 2016 to 2019, and the number of "bad" condition ties increases in the same time period.

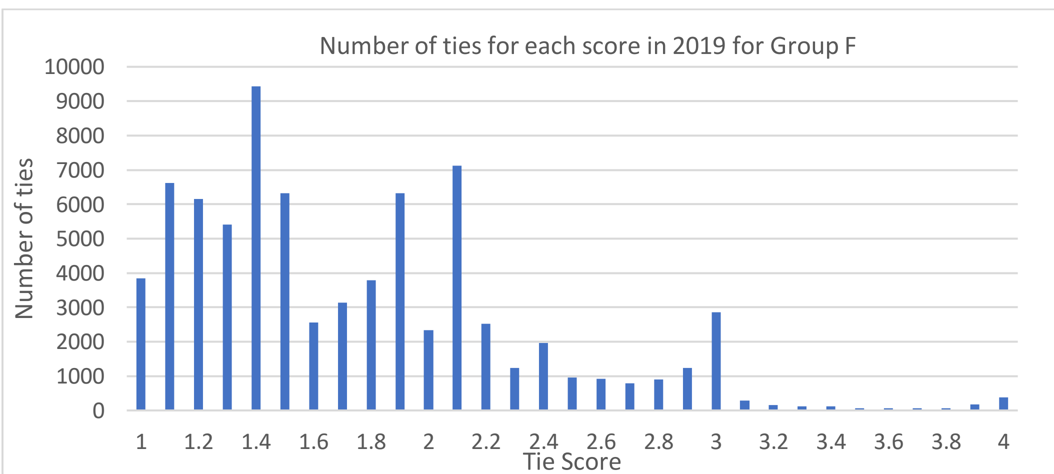
The histograms presented in Figures 20 to 23 show the distributions of ties scores on a decimal level for each group for both 2016 and 2019. Note; moving forward tie condition analysis is based on the decimal scores for each individual tie, as opposed to the digital score discussed previously.

5.2.1 Group F

Figures 20 A and B represent the tie score distributions in group F in 2016 and 2019 respectively.



A: Tie score distribution in 2016 for group F



B: Tie score distribution in 2019 for group F

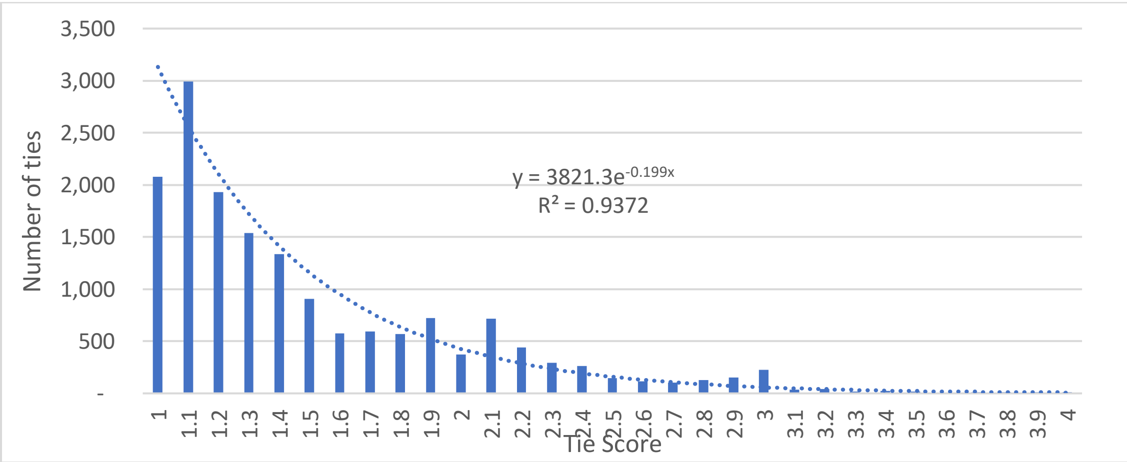
Figure 20: Group F tie score distribution

It can be seen that the distribution of tie scores in 2016 is exponential for group F, while it is not in 2019. As Group F represents the ties having all four adjacent ties in good condition, F will be the reference group to compare how the adjacent tie condition affects the ties scores.

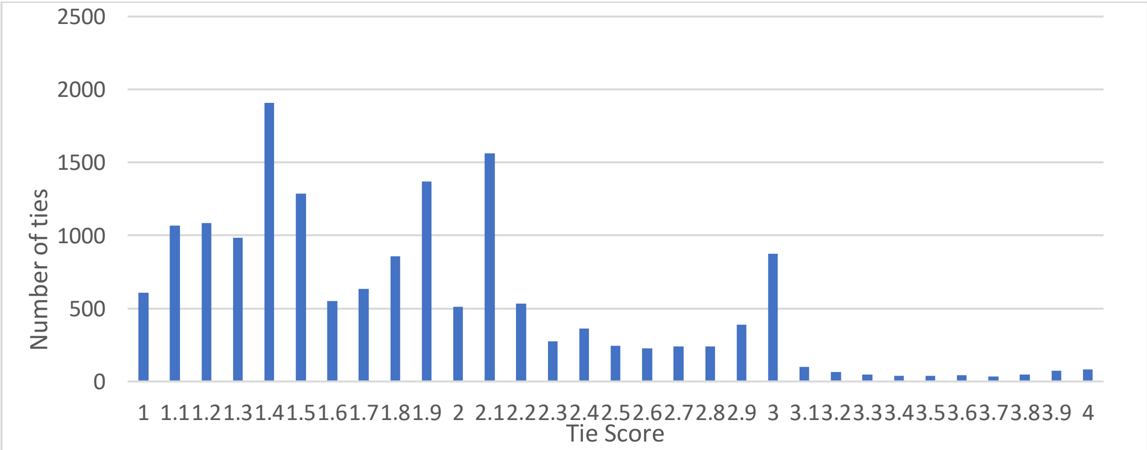
A change in distributions can be noted between 2016 and 2019, such that more ties have a higher tie score in 2019. That indicates an overall degradation of ties between 2016 and 2019 for group F, as would be expected due to normal aging and traffic loading, without significant tie replacement.

5.2.2 Group A

Figures 21 A and B represent the tie score distributions in group A in 2016 and 2019 respectively.



A: Tie score distribution in 2016 for group A



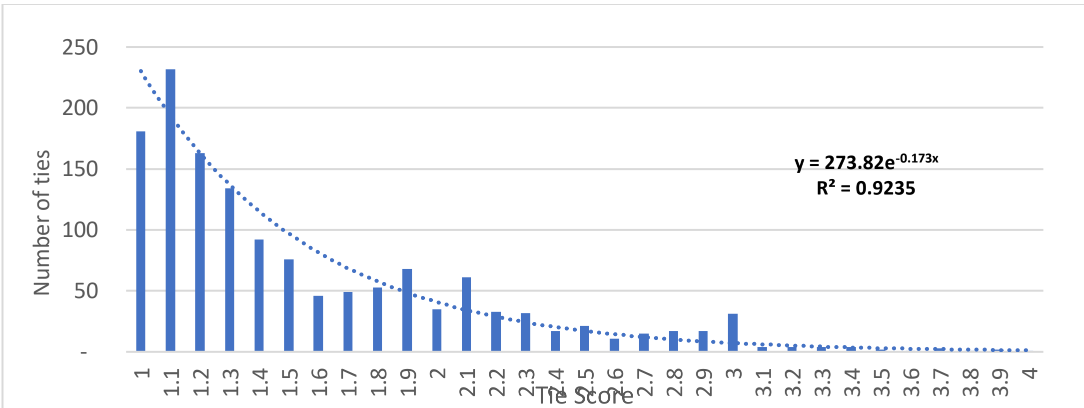
B: Tie score distribution in 2019 for group A

Figure 21: Group A tie score distribution

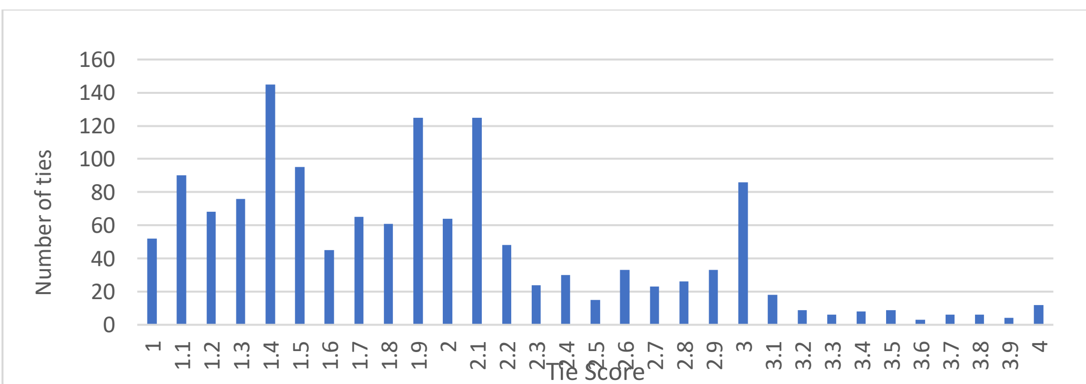
The distribution of tie scores in 2016 is exponential having more ties with scores lower than 1.5. A change in distributions can be noted between 2016 and 2019, such that more ties have a higher tie score in 2019 compared to 2016. That indicates an overall degradation of ties between 2016 and 2019 for group F, as would be expected due to normal aging and traffic loading, without significant tie replacement.

5.2.3. Group B

Figures 22 A and B represent the tie score distributions in group B in 2016 and 2019 respectively.



A: Tie score distribution in 2016 for group B



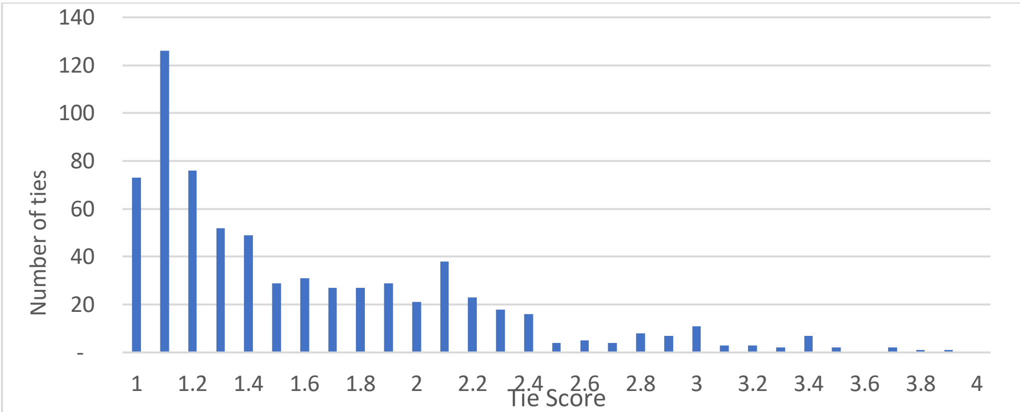
B: Tie score distribution in 2019 for group B

Figure 22: Group B tie score distribution

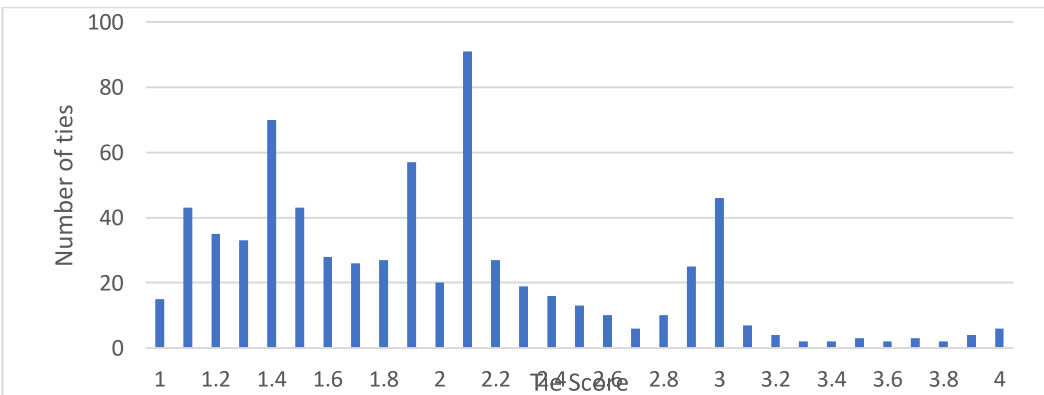
The distribution of tie scores in 2016 is exponential having more ties with scores lower than 1.8. A change in distributions can be noted between 2016 and 2019, such that more ties have a higher tie score in 2019 compared to 2016. That indicates an overall degradation of ties between 2016 and 2019 for group F, as would be expected due to normal aging and traffic loading, without significant tie replacement.

5.2.4. Group C

Figures 23 A and B represent the tie score distributions in group c in 2016 and 2019 respectively.



A: Tie score distribution in 2016 for group C



B: Tie score distribution in 2019 for group C

Figure 23: Group C tie score distribution

The distribution of tie scores in 2016 is exponential having more ties with scores lower than 1.6. A change in distributions can be noted between 2016 and 2019, such that more ties have a higher tie score in 2019 compared to 2016. That indicates an overall degradation of ties between 2016 and 2019 for group F, as would be expected due to normal aging and traffic loading, without significant tie replacement.

5.2.5. Comparison

From the histograms presented in Figures 20-23, a change in distribution was noted for all groups indicating an overall tie condition degradation. In order to quantify this overall condition degradation and compare the effect of loss of support on the degradation of ties, a deeper analysis was performed. Table 14 summarizes the mean, median, and the standard deviation of the tie scores for both 2016 and 2019 as well as their difference for each group.

Table 14: Mean, Median, and Standard Deviation for tie score distributions in 2016 and 2019

	Group	Tie Scores in 2016	Tie Scores in 2019	Difference
Mean	F	1.425	1.748	0.3219
	A	1.514	1.852	0.3369
	B	1.572	1.932	0.36042
	C	1.604	1.964	0.36043
Median	F	1.3	1.6	0.300
	A	1.3	1.8	0.500
	B	1.3	1.9	0.600
	C	1.4	1.9	0.500
Standard Deviation	F	0.4552	0.5840	0.129
	A	0.5269	0.6356	0.109
	B	0.5848	0.6769	0.092
	C	0.5974	0.6653	0.068

The median tie score in 2019 for group F is 1.6 while it is 1.8 for group A and 1.9 for both B and C, suggesting a higher number of degraded ties in the groups with a loss of support.

Also, a higher difference in means between 2016 and 2019 suggests a higher number of degraded ties in the 3 years. For group F (group with 0% loss of support), the means difference is 0.32, while for group C (46% average loss of support), it is 0.36. It can also be noticed that the difference in means between 2016 and 2019 gets higher as the loss of supports gets larger. This indicates again that the greater the loss of support, the faster the degradation of ties.

5.3 Effect Size

Effect size is a measure of the strength of the relationship between two variables in a statistical population. As such it is a way of quantifying the difference between two groups that emphasizes the size of the difference. Alternately, it can be defined as “a quantitative measure of the magnitude of the experimenter effect. The larger the effect size the stronger the relationship between two variables.” [17] In this case, the magnitude of different amounts of loss of support effect (for different groups, since each group represents a different loss of support value) is measured between 2016 and 2019. This effect factor needs to be quantified to see how it changes as the loss of support changes from Group F to Group C.

Three effect size methods were used to compare the different groups of ties (different loss of supports) between 2016 and 2019. Group F served as the comparison reference because it represents the group of ties with good adjacent tie condition and no loss of adjacent tie support.

5.3.1. Cohen's d effect size

"Cohen's d is an appropriate effect size for the comparison between two means" [17]

Mathematically:

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s} \quad \text{such that:} \quad s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2}} \qquad [18]$$

n_1 is the number of elements in population 1 (in this case Group F)
n_2 is the number of elements in population 2 (Group A, B, or C)
s_1 is the standard deviation for population 1 (in this case Group F)
s_2 is the standard deviation for population 2 (Group A, B, or C)
$\bar{x}_1$ mean for population 1 (in this case Group F)
$\bar{x}_2$ mean for population 2 (Group A, B, or C)

F is the reference, as it represents the best case scenario (all adjacent ties in good condition) and it is necessary to measure how the change of adjacent tie condition affects each Group within 3 years. The means and standards deviations used are the ones in 2019. Table 15 shows the effect size using Cohen's method.

Table 15: Cohen's effect size

	Effect Size with F as a reference	
	s	**d**
A	0.593238	0.17545
B	0.585732	0.315255
C	0.584722	0.370975

Taking the reference population as F, the effect size for A is 0.18, while it is 0.22 for B and 0.37 for C. The effect size gets higher as the loss of supports increases.

5.3.2. Glass's Δ method of effect size

"This method is similar to the Cohen's method, but in this method standard deviation is used for the second group"[18]. Mathematically this formula can be written as:

$$\Delta = \frac{\bar{x}_1 - \bar{x}_2}{s_2} \qquad [18]$$

Such that
s_2 is the standard deviation for population 2 (in this case Group F)

$\bar{x}_1$ mean for population 1 (Group A, B, or C)
$\bar{x}_2$ mean for population 2 (in this case Group F)

Table 16 below represents the Δ effect size using Glass's method to compare F and A, F and B, as well as F and C respectively:

Table 16: Glass's effect size

Tier	Population 2 = F	Population 1 = F
A	0.18	-0.16377
B	0.32	-0.27281
C	0.37	-0.32604

Taking population 2 as F, the effect size's absolute value for A is 0.16, while it is 0.27 for B and 0.33 for C. The absolute value of Δ effect size gets higher as the loss of support increases.

5.3.3. Edges' g Method of Effect Size

Another method of effect size to compare Group F to all 3 tiers is Edges' g method, where the effect size factor g can be computed as follow:

$$g = \frac{\bar{x}_1 - \bar{x}_2}{s^*}$$ [18]

Such that s is the standard deviation, and
$\bar{x}_1$ mean for population 1 (Group A, B, or C)
$\bar{x}_2$ mean for population 2 (in this case Group F)

Table 17 shows the effect size using Edges' method.

Table 17: Edges' effect Size

	Effect Size with F as a reference	
Groups	s	g
A	0.468461	**0.222182**
B	0.457852	**0.403308**
C	0.456681	**0.474986**

Taking the reference population as F, the effect size for A is 0.18, while it is 0.22 for B and 0.37 for C. The effect size g gets higher as the support condition decreases (from Group A to Group C).

5.3.4. Conclusions from the Effect Size analysis

As noted, the effect size is a statistical measure that allowed for the quantification of the effect that a particular “process” has on different populations. The higher the effect size factor, the stronger the effect the “process” has on a particular population.
By defining the different loss of support as different “processes” and comparing the effect size based on the distribution of scores, the effect of different adjacent tie support conditions can be quantified. Based on the results of three such Edge Effect analyses, it can be concluded that the higher the loss of support the higher the effect size, and hence the higher the rate of tie degradation.

5. 4. Tie Condition Changes Within Three Years

The different inspections in 2016 as well as 2019 were accurately aligned to allow for a direct tie by tie comparison of condition at the different inspection times, as described in section IV (Tie Alignment). This then allowed for the analysis of the change in individual tie condition for all the study ties. Figures 24, 25, 26, and 27 represent the detailed number of ties with different tie score transitions from year 2016 to year 2019 for each tie support group.

Score in 2019

Score in 2016	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	2,012	2,261	1,477	973	1,705	814	396	442	451	592	271	468	138	60	98	56	51	39	43	36	45	17	12	18	7	12	8	11	8	15	87
1.1	-	2,843	1,898	1,283	2,671	1,335	621	697	792	1,029	336	844	239	75	147	65	60	41	40	42	40	17	13	12	11	12	13	14	11	16	121
1.2	-	-	1,259	891	1,801	1,001	489	597	657	1,003	313	867	236	72	144	77	65	34	41	44	45	14	11	6	6	12	7	9	8	11	79
1.3	-	-	-	700	1,383	768	454	519	672	1,034	294	847	215	81	96	80	66	36	52	39	46	16	11	9	14	7	6	9	9	10	62
1.4	-	-	-	-	1,050	668	416	540	608	1,085	385	868	200	101	132	70	51	52	47	32	49	12	17	10	8	5	4	16	5	17	66
1.5	-	-	-	-	-	526	309	401	513	831	252	867	227	92	176	79	66	57	59	45	64	16	12	12	6	3	9	10	7	10	51
1.6	-	-	-	-	-	-	138	187	234	448	106	660	232	67	137	67	45	51	43	35	57	13	10	6	6	6	3	3	2	7	27
1.7	-	-	-	-	-	-	-	141	205	426	121	652	242	63	185	101	57	47	61	75	65	13	10	9	7	7	3	6	3	7	27
1.8	-	-	-	-	-	-	-	-	198	455	122	680	264	131	234	84	107	59	94	96	121	14	15	9	11	7	6	8	7	7	37
1.9	-	-	-	-	-	-	-	-	-	479	186	742	304	133	175	130	123	101	95	140	165	21	19	10	5	8	14	11	9	13	57
2	-	-	-	-	-	-	-	-	-	-	177	340	123	91	94	71	94	75	59	74	112	23	3	13	8	11	9	7	8	7	23
2.1	-	-	-	-	-	-	-	-	-	-	-	493	221	123	197	147	154	159	135	256	412	29	20	9	20	6	14	7	15	20	61
2.2	-	-	-	-	-	-	-	-	-	-	-	-	157	104	162	109	109	107	145	229	450	20	21	20	13	16	5	10	13	20	43
2.3	-	-	-	-	-	-	-	-	-	-	-	-	-	59	103	75	68	83	123	138	408	14	16	10	16	6	13	16	8	14	54
2.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	64	51	60	66	75	119	378	35	29	17	12	11	21	11	12	25	54
2.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	25	29	32	32	52	185	18	9	6	11	4	8	8	10	11	33
2.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	21	22	29	33	135	14	7	8	6	7	10	7	7	5	22
2.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	20	19	28	128	11	5	7	11	7	7	8	7	16	22
2.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	18	30	137	19	9	9	7	10	11	5	8	14	33
2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	41	152	27	9	10	12	14	11	10	11	13	56
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	222	38	19	17	15	20	18	12	28	52	120
3.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	4	4	4	6	3	4	-	10	17
3.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	4	4	1	1	3	6	6	24
3.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	3	7	2	3	2	3	24
3.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	6	2	4	4	6	27
3.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	3	4	4	3	16
3.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	2	2	4	9
3.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	2	2	11
3.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	5	10
3.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	24
4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	27

Figure 24: Tie Score Changes for Group F

Score in 2019

Score in 2016	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	308	330	195	134	330	152	67	49	85	135	35	106	28	13	13	12	10	6	15	10	11	1	2	6	2	6	2	3	1	2	12
1.1	-	476	304	215	536	264	136	156	173	219	67	211	42	21	25	14	23	18	8	11	20	5	4	4	5	5	2	3	2	5	17
1.2	-	-	206	150	325	199	94	103	169	205	69	188	50	20	26	15	10	10	17	14	16	7	4	5	2	3	-	1	1	1	19
1.3	-	-	-	124	271	159	75	108	141	218	72	153	33	18	28	24	10	11	15	17	9	6	5	1	-	4	2	1	5	2	24
1.4	-	-	-	-	215	136	69	98	158	200	90	178	49	10	22	12	11	15	5	16	19	5	5	-	-	1	1	2	1	-	15
1.5	-	-	-	-	-	105	60	79	86	183	51	146	36	21	23	15	16	4	8	21	19	4	1	7	-	3	-	3	1	-	14
1.6	-	-	-	-	-	-	25	41	36	86	35	146	39	12	39	28	16	8	11	13	15	2	1	2	2	-	1	1	1	4	10
1.7	-	-	-	-	-	-	-	40	48	107	23	147	54	23	46	16	15	15	11	14	13	6	-	2	-	1	1	1	1	-	11
1.8	-	-	-	-	-	-	-	-	34	95	20	141	49	28	42	30	22	16	14	20	34	4	2	2	2	1	-	-	-	5	9
1.9	-	-	-	-	-	-	-	-	-	121	66	171	89	32	27	22	26	22	26	39	38	9	4	-	5	3	5	-	-	1	17
2	-	-	-	-	-	-	-	-	-	-	37	95	27	27	14	22	22	24	20	29	27	4	3	4	-	1	1	1	1	3	9
2.1	-	-	-	-	-	-	-	-	-	-	-	134	84	24	53	25	41	39	52	73	120	7	12	7	8	6	7	1	4	5	16
2.2	-	-	-	-	-	-	-	-	-	-	-	-	37	22	31	22	20	44	34	55	129	6	3	4	4	1	4	5	2	1	19
2.3	-	-	-	-	-	-	-	-	-	-	-	-	-	5	27	25	19	12	23	29	108	4	3	-	4	4	3	3	4	7	15
2.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11	13	12	17	16	28	116	7	5	-	3	-	3	1	1	3	26
2.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	7	13	10	27	50	6	4	3	2	3	1	5	3	2	6
2.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	7	7	10	17	40	4	2	2	-	5	2	-	8	3	10
2.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	5	16	44	5	3	2	2	4	2	5	2	2	10
2.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	12	58	7	4	3	6	1	2	3	-	6	18
2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	16	75	7	3	4	2	1	3	4	5	7	22
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	94	19	7	5	3	4	8	3	8	20	53
3.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	5	2	2	4	1	3	5	12
3.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	4	1	1	1	4	1	4	19
3.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	2	-	3	3	-	10
3.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	-	-	2	7	10
3.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	2	-	2	1	9
3.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	2	3
3.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	1	1	6
3.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	3	8
3.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	12
4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	11

Figure 25: Tie Score Changes for Group A

Score in 2019

Score in 2016	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	29	32	18	15	22	9	5	6	8	10	9	5	3	2	2	-	3	1	2	1	1	1	-	1	-	1	-	1	-	-	5
1.1		40	17	11	49	21	14	14	10	29	6	15	3	-	3	-	2	1	-	-	2	-	-	1	-	1	1	1	2	1	6
1.2			11	13	24	17	6	18	18	17	11	15	5	-	5	-	1	-	2	1	-	-	-	1	-	-	1	-	-	1	5
1.3				13	24	13	9	8	11	13	8	16	5	1	2	1	1	-	1	-	3	1	-	-	-	-	1	1	1	1	5
1.4					16	11	3	7	2	17	8	13	2	-	-	1	2	-	3	1	-	-	-	-	1	-	-	-	-	-	2
1.5						11	5	7	3	10	7	11	3	1	1	-	3	2	1	1	-	-	-	-	-	1	-	-	-	-	3
1.6							3	6	2	7	1	8	5	-	1	-	-	1	-	1	3	1	-	-	-	1	-	-	-	-	-
1.7								2	6	8	-	8	6	-	1	-	3	1	1	1	3	1	1	-	-	1	-	-	-	-	-
1.8									1	11	5	9	3	5	-	2	-	2	5	1	1	-	-	-	-	-	-	-	-	-	3
1.9										14	6	11	5	3	5	1	7	3	1	2	5	2	-	-	-	-	-	-	-	-	1
2											2	9	1	1	2	1	3	2	-	-	3	-	-	-	-	-	-	-	1	-	1
2.1												13	6	3	2	1	2	3	5	9	9	1	1	-	-	-	-	-	1	2	1
2.2													1	2	2	2	1	2	-	5	5	1	-	1	1	-	-	1	1	-	-
2.3														1	-	2	2	1	1	3	8	-	-	-	1	-	1	1	1	-	1
2.4															-	2	-	-	-	-	1	1	-	-	-	-	-	1	-	-	2
2.5																-	1	2	1	3	6	1	3	-	-	-	-	-	1	-	2
2.6																	1	-	-	-	3	-	-	-	1	1	-	-	-	-	1
2.7																		-	2	1	8	1	-	-	-	-	-	-	1	1	-
2.8																			1	1	11	-	-	-	-	-	-	-	-	1	2
2.9																				2	8	-	1	-	-	1	-	-	-	2	2
3																					14	5	1	-	1	-	1	5	1	1	5
3.1																						-	-	-	-	-	-	-	1	1	-
3.2																							-	-	-	1	-	-	-	-	1
3.3																								-	-	-	-	-	1	-	1
3.4																									-	1	-	1	-	-	-
3.5																										-	-	-	-	-	1
3.6																											-	-	-	-	2
3.7																												-	-	-	3
3.8																													-	-	1
3.9																														-	2
4																															6

Figure 26: Tie Score Changes for Group B

	Score in 2019																														
Score in 2016	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	5	13	5	5	10	5	2	4	2	3	-	5	1	-	-	2	2	1	-	1	1	-	-	-	-	-	-	-	1	-	3
1.1		22	8	6	26	10	9	4	5	10	2	8	3	2	1	1	1	-	1	2	-	-	1	1	-	-	-	-	1	-	1
1.2			12	5	10	3	3	4	3	11	2	11	2	1	1	1	-	-	-	-	1	-	-	-	-	-	-	-	-	1	2
1.3				3	7	6	4	-	3	4	1	12	1	1	1	-	-	-	-	3	3	-	-	-	1	-	-	-	-	-	-
1.4					9	6	3	2	4	5	1	10	1	-	-	-	-	-	-	2	-	1	-	1	-	-	1	-	-	-	2
1.5						4	-	3	1	6	-	5	5	-	-	1	-	-	-	3	-	1	-	-	-	-	-	-	-	-	-
1.6							1	2	4	2	2	11	-	1	-	-	-	-	1	5	-	-	-	-	-	-	-	-	-	-	-
1.7								3	-	4	1	8	4	-	4	-	-	-	-	1	1	-	-	-	-	-	-	-	-	-	1
1.8									-	4	3	6	-	4	-	6	1	-	-	-	1	1	-	-	-	-	-	-	-	-	-
1.9										6	-	6	3	3	1	1	3	-	1	1	1	1	-	-	1	-	-	-	-	-	-
2											4	1	1	1	-	1	-	1	4	-	1	-	-	-	-	-	-	-	-	-	4
2.1												13	2	1	1	1	1	4	2	4	5	1	-	-	-	-	-	-	-	-	2
2.2													1	1	-	-	1	1	3	1	9	-	-	-	-	1	-	-	1	-	1
2.3														-	-	2	-	-	2	3	9	-	2	-	-	-	-	-	2	-	-
2.4															2	-	2	-	-	2	6	-	2	-	-	-	-	-	2	2	-
2.5																-	-	2	-	2	-	-	-	-	-	-	-	-	-	-	-
2.6																	-	-	-	2	3	-	-	-	-	-	-	-	-	-	-
2.7																		-	-	-	4	-	-	-	-	-	-	-	-	-	-
2.8																			-	-	7	1	-	-	-	-	-	-	-	-	-
2.9																				1	4	-	-	-	-	-	-	-	1	-	-
3																					5	-	-	-	1	-	-	-	-	1	4
3.1																						-	-	-	-	-	-	-	-	-	3
3.2																							-	3	-	-	-	-	-	-	-
3.3																								-	-	-	1	-	-	-	1
3.4																									-	-	-	1	-	3	3
3.5																										-	-	-	1	-	1
3.6																											-	-	-	-	-
3.7																												-	1	1	-
3.8																													-	-	1
3.9																														-	1
4																															-

Figure 27: Tie Score Changes for Group C

It should be noted that 80.8 % of ties belong to group F, where all four adjacent ties are in good condition.

The percentage of ties changing conditions (from their initial condition score) within the 3 years (2016- 2019) is calculated as follows:

$$Percentage = \frac{Number\ of\ ties\ with\ final\ Score\ F\ and\ initial\ score\ I}{Total\ number\ of\ ties\ with\ same\ initial\ score\ I} \qquad \text{Equation 2}$$

where F is the final tie score (2019) and I is the initial score (2016).

The percentages calculated using Equation 2 for tie scores 1.0 to 4.0 in groups F, A, B, and C are presented in Figure 28, 29, 30, and 31. They represent the percentage of ties that change from a particular score in 2016(vertical) to another (higher/more degraded) score in 2019 (horizontal).

	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	15.94%	17.93%	11.70%	7.71%	13.51%	6.45%	3.14%	3.50%	3.57%	4.69%	2.15%	3.71%	1.09%	0.48%	0.78%	0.44%	0.40%	0.31%	0.34%	0.29%	0.36%	0.13%	0.10%	0.14%	0.06%	0.10%	0.06%	0.09%	0.06%	0.12%	0.69%
1.1		18.54%	12.37%	8.36%	17.41%	8.70%	4.05%	4.54%	5.16%	6.71%	2.19%	5.50%	1.56%	0.49%	0.96%	0.42%	0.39%	0.27%	0.26%	0.27%	0.26%	0.11%	0.08%	0.08%	0.07%	0.08%	0.08%	0.09%	0.07%	0.10%	0.79%
1.2			12.87%	9.11%	18.42%	10.24%	5.00%	6.10%	6.72%	10.26%	3.20%	8.87%	2.21%	0.74%	1.47%	0.79%	0.66%	0.35%	0.42%	0.45%	0.46%	0.14%	0.11%	0.06%	0.06%	0.12%	0.07%	0.09%	0.08%	0.11%	0.81%
1.3				9.29%	18.35%	10.19%	6.03%	6.89%	8.92%	13.72%	3.90%	11.24%	2.85%	1.07%	1.27%	1.06%	0.88%	0.48%	0.69%	0.52%	0.61%	0.21%	0.15%	0.12%	0.19%	0.09%	0.08%	0.12%	0.12%	0.13%	0.82%
1.4					16.12%	10.25%	6.39%	8.29%	9.33%	16.66%	5.91%	13.33%	3.07%	1.55%	2.03%	1.07%	0.78%	0.80%	0.72%	0.49%	0.75%	0.18%	0.26%	0.15%	0.12%	0.08%	0.06%	0.25%	0.08%	0.26%	1.01%
1.5						11.19%	6.57%	8.53%	10.91%	17.68%	5.36%	18.45%	4.83%	1.96%	3.74%	1.68%	1.40%	1.21%	1.26%	0.96%	1.36%	0.34%	0.26%	0.26%	0.13%	0.06%	0.19%	0.21%	0.15%	0.21%	1.09%
1.6							5.33%	7.22%	9.03%	17.30%	4.09%	25.48%	8.96%	2.59%	5.29%	2.59%	1.74%	1.97%	1.66%	1.35%	2.20%	0.50%	0.39%	0.23%	0.23%	0.23%	0.12%	0.12%	0.08%	0.27%	1.04%
1.7								5.57%	8.09%	16.82%	4.78%	25.74%	9.55%	2.49%	7.30%	3.99%	2.23%	1.86%	2.41%	2.96%	2.57%	0.51%	0.39%	0.36%	0.28%	0.28%	0.12%	0.24%	0.12%	0.28%	1.07%
1.8									7.16%	16.45%	4.41%	24.58%	9.54%	4.74%	8.10%	3.04%	3.87%	2.13%	3.40%	3.47%	4.37%	0.87%	0.54%	0.33%	0.40%	0.25%	0.22%	0.29%	0.25%	0.25%	1.34%
1.9										16.29%	6.32%	25.23%	10.34%	4.52%	5.95%	4.42%	4.18%	3.43%	3.23%	4.76%	5.61%	0.75%	0.65%	0.34%	0.17%	0.27%	0.48%	0.37%	0.31%	0.44%	1.94%
2											12.45%	23.91%	8.65%	6.40%	6.61%	4.99%	6.61%	5.27%	4.15%	5.20%	7.88%	1.62%	0.21%	0.91%	0.56%	0.77%	0.63%	0.49%	0.56%	0.49%	1.62%
2.1												19.74%	8.85%	4.92%	7.89%	5.88%	6.16%	6.37%	5.40%	10.25%	16.49%	1.16%	0.80%	0.36%	0.80%	0.24%	0.56%	0.28%	0.60%	0.80%	2.44%
2.2													8.96%	5.93%	9.24%	6.22%	6.22%	6.10%	8.27%	13.06%	25.67%	1.14%	1.20%	1.14%	0.74%	0.91%	0.29%	0.57%	0.74%	1.14%	2.45%
2.3														4.78%	8.35%	6.08%	5.51%	6.73%	9.97%	11.18%	33.06%	1.94%	1.30%	0.81%	1.30%	0.49%	1.05%	1.30%	0.65%	1.13%	4.38%
2.4															6.21%	4.95%	5.83%	6.41%	7.28%	11.55%	36.70%	3.40%	2.82%	1.65%	1.17%	1.07%	2.04%	1.07%	1.17%	1.46%	5.24%
2.5																5.29%	6.13%	6.77%	6.77%	10.99%	39.11%	3.81%	1.90%	1.27%	2.33%	0.85%	1.69%	1.69%	2.11%	2.33%	6.98%
2.6																	7.01%	7.01%	9.24%	10.51%	36.62%	4.46%	2.23%	2.55%	1.91%	2.23%	3.18%	2.23%	2.23%	1.59%	7.01%
2.7																		6.76%	6.42%	9.46%	43.24%	3.72%	1.69%	2.36%	3.72%	2.36%	2.36%	2.70%	2.36%	5.41%	7.43%
2.8																			5.81%	9.68%	44.19%	6.13%	2.90%	2.90%	2.26%	3.23%	3.55%	1.61%	2.58%	4.52%	10.65%
2.9																				10.10%	47.29%	6.65%	2.22%	2.46%	2.96%	3.45%	2.71%	2.46%	2.71%	3.20%	13.79%
3																					41.04%	7.02%	3.51%	3.14%	2.77%	3.70%	3.33%	2.22%	5.18%	5.91%	22.18%
3.1																						1.89%	7.55%	7.55%	7.55%	11.32%	5.66%	7.55%	0.00%	18.87%	32.08%
3.2																							2.00%	8.00%	8.00%	2.00%	2.00%	6.00%	12.00%	12.00%	48.00%
3.3																								6.38%	6.38%	14.89%	4.26%	6.38%	4.26%	6.38%	51.06%
3.4																									2.00%	12.00%	4.00%	8.00%	8.00%	12.00%	54.00%
3.5																										6.25%	9.38%	12.50%	12.50%	9.38%	50.00%
3.6																											10.53%	10.53%	10.53%	21.05%	47.37%
3.7																												16.67%	11.11%	11.11%	61.11%
3.8																													6.25%	31.25%	62.50%
3.9																														17.24%	82.76%
4																															100.00%

Figure 28: Percentage of tie score changes for Group F

	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	14.80%	15.86%	9.37%	6.44%	15.86%	7.30%	3.22%	2.35%	4.08%	6.49%	1.68%	5.09%	1.35%	0.62%	0.62%	0.58%	0.48%	0.29%	0.72%	0.48%	0.53%	0.05%	0.10%	0.29%	0.10%	0.29%	0.10%	0.14%	0.05%	0.10%	0.58%
1.1		15.91%	10.16%	7.19%	17.92%	8.83%	4.55%	5.22%	5.78%	7.32%	2.24%	7.05%	1.40%	0.70%	0.84%	0.47%	0.77%	0.60%	0.27%	0.37%	0.67%	0.17%	0.13%	0.13%	0.17%	0.17%	0.07%	0.10%	0.07%	0.17%	0.57%
1.2			10.68%	7.78%	16.85%	10.32%	4.87%	5.34%	8.76%	10.63%	3.58%	9.75%	2.59%	1.04%	1.35%	0.78%	0.52%	0.52%	0.88%	0.73%	0.83%	0.36%	0.21%	0.26%	0.10%	0.16%	0.00%	0.05%	0.05%	0.05%	0.98%
1.3				8.07%	17.64%	10.35%	4.88%	7.03%	9.18%	14.19%	4.69%	9.96%	2.15%	1.17%	1.82%	1.56%	0.65%	0.72%	0.98%	1.11%	0.59%	0.39%	0.33%	0.07%	0.00%	0.26%	0.13%	0.07%	0.33%	0.13%	1.56%
1.4					16.13%	10.20%	5.18%	7.35%	11.85%	15.00%	6.75%	13.35%	3.68%	0.75%	1.65%	0.90%	0.83%	1.13%	0.38%	1.20%	1.43%	0.38%	0.38%	0.00%	0.00%	0.08%	0.08%	0.15%	0.08%	0.00%	1.13%
1.5						11.59%	6.62%	8.72%	9.49%	20.20%	5.63%	16.11%	3.97%	2.32%	2.54%	1.66%	1.77%	0.44%	0.88%	2.32%	2.10%	0.44%	0.11%	0.77%	0.00%	0.33%	0.00%	0.33%	0.11%	0.00%	1.55%
1.6							4.36%	7.14%	6.27%	14.98%	6.10%	25.44%	6.79%	2.09%	6.79%	4.88%	2.79%	1.39%	1.92%	2.26%	2.61%	0.35%	0.17%	0.35%	0.35%	0.00%	0.17%	0.17%	0.17%	0.70%	1.74%
1.7								6.72%	8.07%	17.98%	3.87%	24.71%	9.08%	3.87%	7.73%	2.69%	2.52%	2.52%	1.85%	2.35%	2.18%	1.01%	0.00%	0.34%	0.00%	0.17%	0.17%	0.17%	0.17%	0.00%	1.85%
1.8									5.96%	16.67%	3.51%	24.74%	8.60%	4.91%	7.37%	5.26%	3.86%	2.81%	2.46%	3.51%	5.96%	0.70%	0.35%	0.35%	0.35%	0.18%	0.00%	0.00%	0.00%	0.88%	1.58%
1.9										16.74%	9.13%	23.65%	12.31%	4.43%	3.73%	3.04%	3.60%	3.04%	3.60%	5.39%	5.26%	1.24%	0.55%	0.00%	0.69%	0.41%	0.69%	0.00%	0.00%	0.14%	2.35%
2											9.97%	25.61%	7.28%	7.28%	3.77%	5.93%	5.93%	6.47%	5.39%	7.82%	7.28%	1.08%	0.81%	1.08%	0.00%	0.27%	0.27%	0.27%	0.27%	0.81%	2.43%
2.1												18.66%	11.70%	3.34%	7.38%	3.48%	5.71%	5.43%	7.24%	10.17%	16.71%	0.97%	1.67%	0.97%	1.11%	0.84%	0.97%	0.14%	0.56%	0.70%	2.23%
2.2													8.35%	4.97%	7.00%	4.97%	4.51%	9.93%	7.67%	12.42%	29.12%	1.35%	0.68%	0.90%	0.90%	0.23%	0.90%	1.13%	0.45%	0.23%	4.29%
2.3														1.69%	9.15%	8.47%	6.44%	4.07%	7.80%	9.83%	36.61%	1.36%	1.02%	0.00%	1.36%	1.36%	1.02%	1.02%	1.36%	2.37%	5.08%
2.4															4.20%	4.96%	4.58%	6.49%	6.11%	10.69%	44.27%	2.67%	1.91%	0.00%	1.15%	0.00%	1.15%	0.38%	0.38%	1.15%	9.92%
2.5																4.05%	4.73%	8.78%	6.76%	18.24%	33.78%	4.05%	2.70%	2.03%	1.35%	2.03%	0.68%	3.38%	2.03%	1.35%	4.05%
2.6																	5.98%	5.98%	8.55%	14.53%	34.19%	3.42%	1.71%	1.71%	0.00%	4.27%	1.71%	0.00%	6.84%	2.56%	8.55%
2.7																		3.77%	4.72%	15.09%	41.51%	4.72%	2.83%	1.89%	1.89%	3.77%	1.89%	4.72%	1.89%	1.89%	9.43%
2.8																			3.23%	9.68%	46.77%	5.65%	3.23%	2.42%	4.84%	0.81%	1.61%	2.42%	0.00%	4.84%	14.52%
2.9																				10.74%	50.34%	4.70%	2.01%	2.68%	1.34%	0.67%	2.01%	2.68%	3.36%	4.70%	14.77%
3																					41.96%	8.48%	3.13%	2.23%	1.34%	1.79%	3.57%	1.34%	3.57%	8.93%	23.66%
3.1																						2.86%	0.00%	14.29%	5.71%	5.71%	11.43%	2.86%	8.57%	14.29%	34.29%
3.2																							10.26%	10.26%	2.56%	2.56%	2.56%	10.26%	2.56%	10.26%	48.72%
3.3																								10.00%	0.00%	10.00%	0.00%	15.00%	15.00%	0.00%	50.00%
3.4																									13.64%	0.00%	0.00%	0.00%	9.09%	31.82%	45.45%
3.5																										12.50%	12.50%	0.00%	12.50%	6.25%	56.25%
3.6																											14.29%	0.00%	14.29%	28.57%	42.86%
3.7																												11.11%	11.11%	11.11%	66.67%
3.8																													8.33%	25.00%	66.67%
3.9																														7.69%	92.31%
4																															100.00%

Figure 29: Percentage of tie score changes for Group A

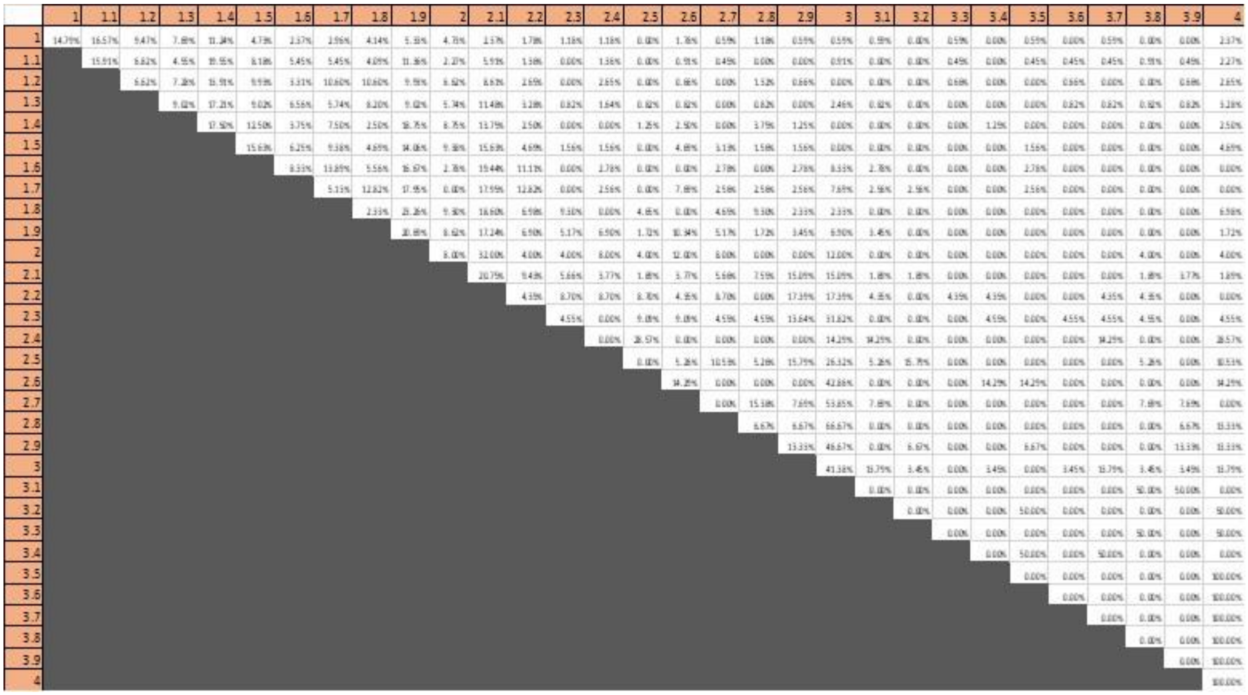


Figure 30: Percentage of tie score changes for Group B

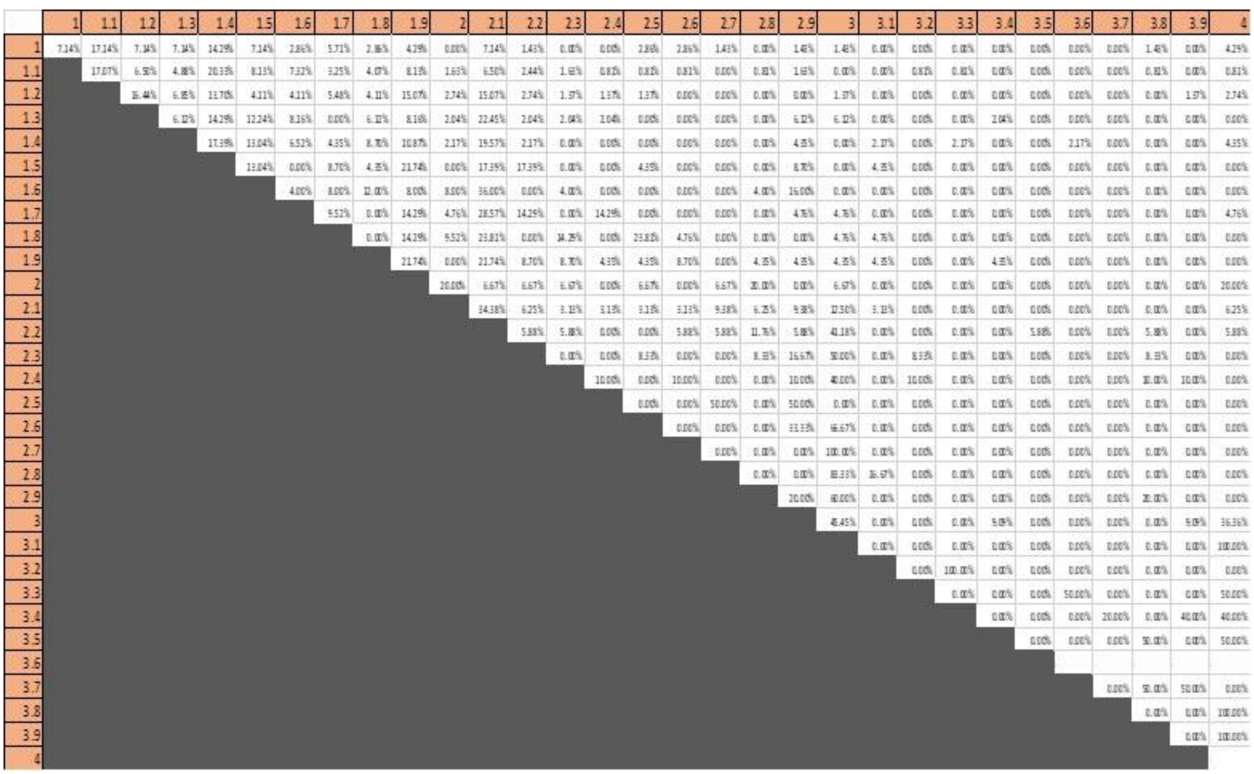


Figure 31: Percentage of tie score changes for Group C

Figures 28-31 show the detailed tie score transitions from year 2016 to year 2019, summarizing the changing tie conditions on a decimal level for groups A, B, C, and F. Appendix A shows the transition for different Tier bundles: Tier A+F, Tier B+C, and Tier A+B+C. It should be noted that the dataset is unbalanced, with tiers B and C having relatively low numbers of ties, while Group F represents about 80% of the dataset.

6. Probability of Tie Failure as a Function of Loss of Support

This section of the report addresses the modeling of tie failure probability using the cleaned and aligned data presented previously. Because of the tie score data distribution, and the unbalance in the dataset, the tie scores were grouped into ranges of 0.5:

- Tie Scores between 1 and 1.4,
- Tie Scores between 1.5 and 1.9,
- Tie Scores between 2 and 2.4,
- Tie Scores between 2.5 and 2.9,
- Tie Scores between 3 and 3.4,
- Tie Scores between 3.5 and 4.

Table 18 summarizes the number of ties in each group and with their initial scores (in 2016):

Table 18: Initial Scores

		Number of Ties in Group			
Initial Score Between		**F**	**A**	**B**	**C**
1	**1.4**	51,937	9,842	849	378
1.5	**1.9**	14,772	3,257	275	144
2	**2.4**	7,196	1,981	149	117
2.5	**2.9**	2,227	685	79	28
3	**3.4**	990	383	42	24
3.5	**4**	815	231	16	4

Note that for group F the loss of support is 0%, for group A, the loss of support is 16.67%, while it is 33% and 46.44% for Group B and C respectively.

Noting that the reported change of tie condition (on a decimal scale) was traced for each individual tie, the data was analyzed based on tie scores ranges of 0.5. Table 19 presents this data based on both initial and final tie condition scores. Thus, for example, the first cell (47.6%) represents the percentage of ties having an initial score between 1 and 1.4 and a final score between 1 and 1.4 and a 0% loss of support. The sum of the values horizontally (100%) in the first line represents the total number of ties with a 0% loss of support (51,937).

Table 19: Percentage of ties depending on their loss of support and final score

		Final Score

Initial Score	Percent Loss of support	1- 1.4	1.5-1.9	2-2.4	2.5-2.9	3-3.4	3.5-4
1- 1.4	0	47.6%	33.6%	14.2%	2.4%	0.9%	1.3%
	16.67	42.7%	35.6%	15.5%	3.2%	1.4%	1.5%
	33	39.4%	34.9%	16.4%	3.2%	1.6%	4.4%
	46.44	39.3%	31.9%	18.0%	4.7%	2.8%	3.3%
1.5- 1.9	0		34.0%	46.1%	12.8%	4.7%	2.5%
	16.67		33.1%	45.2%	13.6%	5.2%	2.9%
	33		35.4%	38.3%	15.4%	6.7%	4.2%
	46.44		28.3%	46.9%	17.7%	6.2%	0.9%
2- 2.4	0			32.5%	35.0%	26.0%	6.5%
	16.67			30.6%	34.2%	27.7%	7.4%
	33			31.5%	33.8%	23.1%	11.5%
	46.44			27.9%	30.2%	29.1%	12.8%
2.5- 2.9	0				23.9%	54.1%	22.0%
	16.67				24.5%	52.6%	22.9%
	33				21.7%	56.5%	21.7%
	46.44				22.2%	72.2%	5.6%
3- 3.4	0					43.3%	56.7%
	16.67					46.5%	53.5%
	33					48.6%	51.4%
	46.44					38.9%	61.1%

It is to be noted that the Initial score is from the 2016 data and the Final score represents the data from 2019.

6.1. Surface Fitting: MATLAB Modeling

In order to determine the probability that a tie will change from a given initial condition value to a given final condition value over the three years, a surface fit[6] was performed on the dataset. A surface fit is a method used to find an equation describing the behavior of two different variables as a function of a third.

It was found that no one equation would define the full range of initial and final tie conditions and the associated support conditions, so different surface fittings were developed, as a function of the initial tie condition or score. These surface fitting generated an appropriate equation describing the degradation behavior of the ties. Thus, for ties having a specific initial score, the equation predicts the probability of a final score Y in 3 years (from 2016 to 2019) given the loss of support X.

The surface fittings were performed for each range of initial score separately using MATLAB. Hence, different equations describe different tie condition behavior, depending on the initial tie condition.

[6] The surface fit was performed using MATLAB.

The surface fittings were performed for each range of initial score separately. Hence, different equations describe different tie condition behavior, depending on the initial tie condition. Table 20 shows the scope of the equations needed to represent the full range of data; as defined by Initial Score (SI) and Final Score (SF). Note, the number of equations required, with some of the ranges requiring more than one equation.

Table 20: Equivalent Equations for each range

Initial Score (SI)		**Final Score (SF)**		**Equation P (SI, Ls, SF)**
1	1.4	1	4	**Equation A**
1.5	1.9	1	2.9	**Equation B1**
		3	3.5	**Equation B2**
		3.6	4	**Equation B3**
2	2.4	2	4	**Equation C**
2.5	2.9	2.5	2.8	**Equation D1**
		2.9	3.4	**Equation D2**
		3.5	4	**Equation D3**
3	3.4	3	4	**Equation E**

The following sections describe and explain the different surface fittings performed for each range of Initial Score (SI) and Final Score (SF) as well as their equivalent resulting equations as follows:

- Section 6.1.1: Equation A
- Section 6.1.2:
    - Section 6.1.2.1: Equation B1
    - Section 6.1.2.2: Equation B2
    - Section 6.1.2.3: Equation B3
- Section 6.1.3: Equation C
- Section 6.1.4: Equation D1, Equation D2, Equation D3
- Section 6.1.5: Equation E

6.1.1 Equation A: Initial Score Between 1 and 1.4

In this section, an equation was generated to model the behavior of ties with an initial score between 1 and 1.4.

Table 21 represents the percentage of ties with an initial score between 1 and 1.4 and their final scores depending on their adjacent tie condition. Initial scores represent score in 2016, while final scores represent scores in 2019.

Table 21: Ties with initial score between 1 and 1.4

Initial Score	**Percent Loss of support**	**Final Score**					
		1-1.4	1.5-1.9	2-2.4	2.5-2.9	3-3.4	3.5-4
1-1.4	0	47.6%	33.6%	14.2%	2.4%	0.9%	1.3%
	16.67	42.7%	35.6%	15.5%	3.2%	1.4%	1.5%
	33	39.4%	34.9%	16.4%	3.2%	1.6%	4.4%
	46.44	39.3%	31.9%	18.0%	4.7%	2.8%	3.3%

Using MATLAB, a surface fit was performed to model the final score, the percent loss of support, and the percentage of ties in each Group. An equivalent equation was generated.

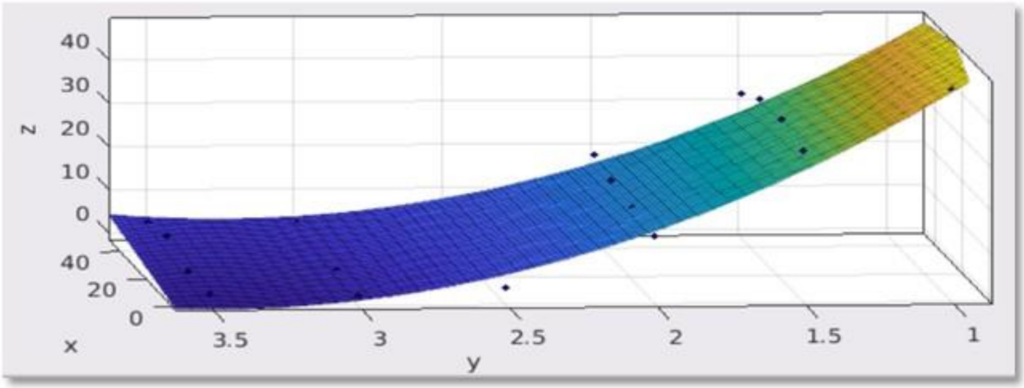


Figure 32: Surface fit for probability of a final score for an initial score between 1 and 1.4

Figure 32 represents the MATLAB surface fit output such that:

- x represents the percent loss of support,
- y represents the final score, and
- z represents the percentage of ties (in the dataset) having a final score of y and a loss of support of x.

The surface resulting equation f(x, y) (Equation A), represents the probability that a tie with initial score (between 1 and 1.4) and a loss of support of x gets a final score of y in 3 years as follows:

Equation A $$f(x,y) = p00 + p10*x + p01*y + p11*x*y + p02*y^2$$

The Coefficients (with 95% confidence bounds) are:

p00 =	94.91
p10 =	-0.179
p01 =	-55.21
p11 =	0.07953
p02 =	7.936

Goodness of fit parameters can be summarized by:

- SSE (sum of squares error): 255
- R-square (representing how accurate the fit is): 0.9594
- Adjusted R-square: 0.9509
- RMSE (Root Mean Square Error): 3.663

6.1.2. Equations B1, B2, and B3: Initial Score between 1.5 and 1.9

In this section, three equations (B1, B2, and B3) were generated to model the behavior of ties with an initial score between 1.5 and 1.9.

Table 22 represents the percentage of ties with an initial score between 1.5 and 1.9 and their final scores depending on their adjacent tie condition. The initial score represents the score in 2016, while the final score represents the score in 2019.

Table 22: Ties with initial score between 1.5 and 1.9

Initial Score	Percent Loss of support	**Final Score**				
		1.5-1.9	**2-2.4**	**2.5-2.9**	**3-3.4**	**3.5-4**
1.5-1.9	0	34.00%	46.10%	12.80%	4.70%	2.50%
	16.67	33.10%	45.20%	13.60%	5.20%	2.90%
	33	35.40%	38.30%	15.40%	6.70%	4.20%
	46.44	28.30%	46.90%	17.70%	6.20%	0.90%

For this particular range of Initial Score, i.e., between 1.5 and 1.9, the modelling was performed depending on the targeted final score:

- Initial Score between 1.5 and 1.9, Final Score Between 1.5 and 2.9, and the resulting equation is: Equation B1,
- Initial Score between 1.5 and 1.9, Final Score Between 3 and 3.5, and the resulting equation is: Equation B2,
- Initial Score between 1.5 and 1.9, Final Score Between 3.6 and 4, and the resulting equation is: Equation B3.

The following sections describe each of the three equations: B1, B2, and B3.

6.1.2.1. Equation B1

Equation B1 describes the behavior of the behavior of ties having an Initial Score between 1.5 and 1.9 and a Final Score between 1.5 and 2.9.

Using MATLAB, a surface fit was performed to model the final score, the percent loss of support and the percentage.

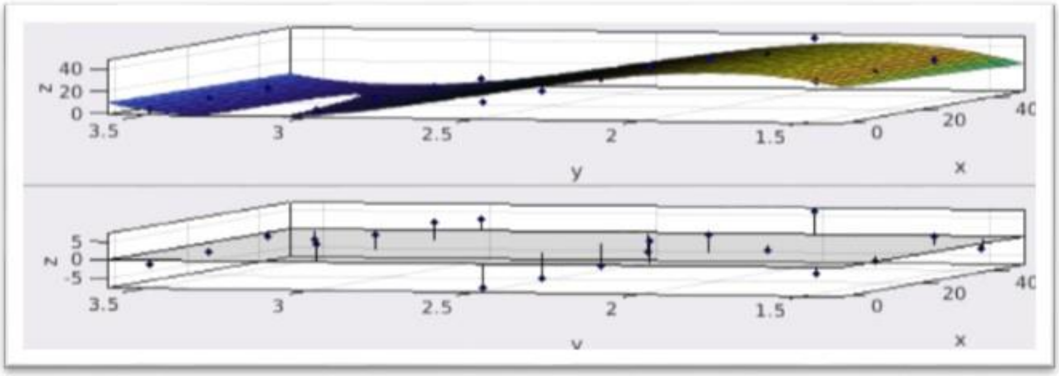


Figure 33: Surface fit for probability of a final score y for an initial score between 1.5 and 1.9 between 1.5 and 1.9 and a Final Score between 1.5 and 2.9

Figure 33 above represents the MATLAB surface fit output such that:

- x represents the percent loss of support,
- y represents the final score, and
- z represents the percentage of ties (in the dataset) having a final score of y and a loss of support of x.

The surface resulting equation f(x, y) (Equation B1) represents the probability that a tie with initial score (1.5 and 1.9) and a loss of support of x gets a final score of y (between 1.5 and 2.9) in 3 years as follows:

Equation B1 f(x,y) = p00 + p10*x + p01*y + p11*x*y + p02*y^2 + p12*x*y^2 + p03*y^3

The Coefficients (with 95% confidence bounds) are:

p00 =	-346.8
p10 =	-0.814
p01 =	535.4
p11 =	0.6566
p02 =	-233.5
p12 =	-0.1226
p03 =	31.18

Goodness of fit parameters can be summarized by:

- SSE: 394.8
- R-square: 0.9241
- Adjusted R-square: 0.8891
- RMSE: 5.511

6.1.2.2. Equation B2

Equation B2 describes the behavior of ties having an Initial Score between 1.5 and 1.9 and a Final Score Between 3 and 3.5.

Using MATLAB, a surface fit was performed to model the final score, the percent loss of support and the percentage of ties in each Group. An equivalent equation was generated.

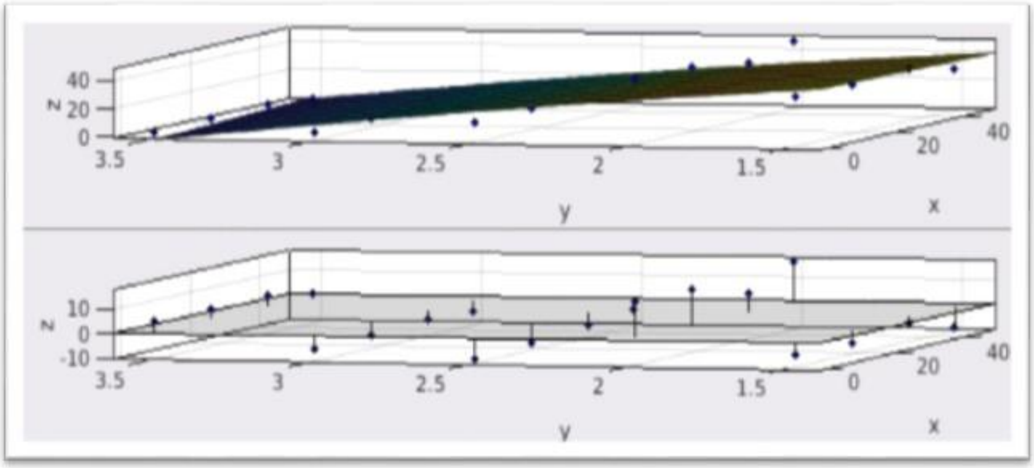


Figure 34: Surface fit for probability of a final score for an initial score between 1.5 and 1.9 and a Final Score Between 3 and 3.5

Figure 34 represents the MATLAB surface fit output such that:

- x represents the percent loss of support,
- y represents the final score, and
- z represents the percentage of ties (in the dataset) having a final score of y and a loss of support of x.
-

The surface resulting equation f(x, y) (Equation B2), represents the probability that a tie with initial score between 1.5 and 1.9 and a loss of support of x gets a final score of y (between 3 and 3.5) in 3 years as follows:

Equation B2 $$f(x,y) = p00 + p10*x + p01*y + p11*x*y + p02*y^2$$

The Coefficients (with 95% confidence bounds) are as follow:

p00 =	***57.25***
p10 =	*-0.1088*
p01 =	*-8.007*
p11 =	*0.04338*
p02 =	*-2.55*

Goodness of fit parameters can be summarized by:

- SSE: 1286
- R-square: 0.7528
- Adjusted R-square: 0.6868
- RMSE: 9.25

6.1.2.3. Equation B3

Equation B3 describes the behavior of ties having an Initial Score between 1.5 and 1.9 and a Final Score Between 3.6 and 4.

In a similar manner to the surface fitting performed for Equation B1, using MATLAB, a surface fit was performed to model the final score, the percent loss of support and the percentage. The equivalent equation was generated as follows:

Equation B3 f(x,y) = p00 + p10*x + p01*y + p11*x*y + p02*y^2 + p12*x*y^2 + p03*y^3

The Coefficients (with 95% confidence bounds) are:

p00 =	-346.8
p10 =	-0.814
p01 =	535.4
p11 =	0.6566
p02 =	-233.5
p12 =	-0.1226
p03 =	31.18

Goodness of fit parameters can be summarized by:

- SSE: 394.8
- R-square: 0.9241
- Adjusted R-square: 0.8891
- RMSE: 5.511

6.1.3. Equation C: Initial Score Between 2 and 2.4

In this section, an equation (Equation C) was generated to model ties with an initial score between 2 and 2.4.

Table 23 represents the percentage of ties with an initial score between 2 and 2.4 and their final scores depending on their adjacent tie condition. The initial score represents the score in 2016, while the final score represents the score in 2019.

Table 23: Ties with initial score between 2 and 2.4

		Final Score			
Initial Score	Percent Loss of support	2-2.4	2.5- 2.9	3-3.4	3.5-4
2-2.4	0	32.50%	35.00%	26.00%	6.50%
	16.67	30.60%	34.20%	27.70%	7.40%
	33	31.50%	33.80%	23.10%	11.50%
	46.44	27.90%	30.20%	29.10%	12.80%

Using MATLAB, a surface fit was performed to model the final score, the percent loss of support and the percentage. An equivalent equation was generated.

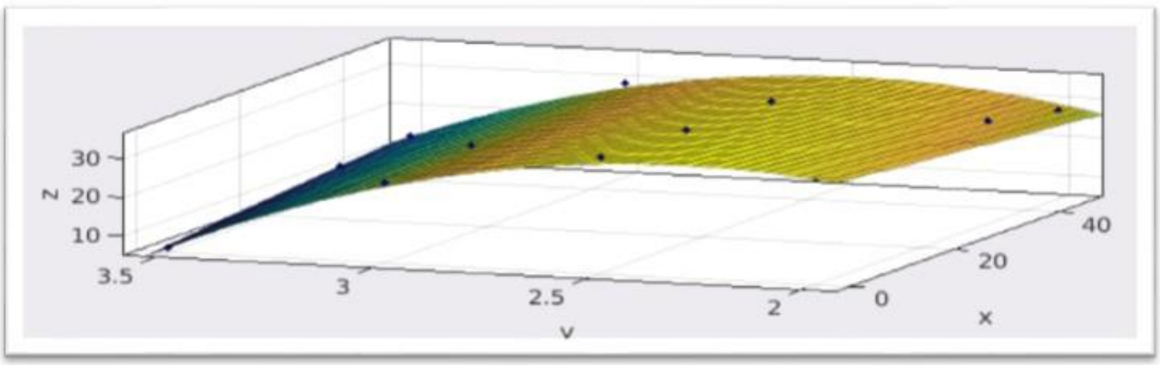


Figure 35 : Surface fit for probability of a final score for an initial score between 2 and 2.4

Figure 35 above represents the MATLAB surface fit output such that:

- x represents the percent loss of support,
- y represents the final score, and
- z represents the percentage of ties (in the dataset) having a final score of y and a loss of support of x.

The surface resulting equation f(x, y) (Equation C), represents the probability that a tie with initial score (between 2 and 2.4) and a loss of support of x gets a final score of y in 3 years as follows:

Equation C $$f(x,y) = p00 + p10*x + p01*y + p11*x*y + p02*y^2$$

The Coefficients (with 95% confidence bounds) are as follow:

p00 =	-67.96
p10 =	-0.4421
p01 =	89.93
p11 =	0.1608
p02 =	-19.6

Goodness of fit parameters can be summarized by:

- SSE: 124.7
- R-square: 0.9742
- Adjusted R-square: 0.9591
- RMSE: 3.3521

6.1.4. Equations D1, D2, and D3: Initial Score between 2.5 and 2.9

In this section, Equations D1, D2, and D3 were generated to model the behavior of ties with an initial score between 2.5 and 2.9.

For ties with an initial score between 2.5 and 2.9, performing an overall surface fit did not lead to satisfactory results. So, as an alternative, the final scores were considered as decimals of ranges of 0.1 rather than 0.5, and a loss of support weighted average was computed, as it can be seen in the last row of Table 24.

Table 24 represents the number of ties with an initial score between 2.5 and 2.9 and their final scores depending on their adjacent tie condition.

Table 24: Number of ties with an initial score between 2.5 and 2.9 and their final scores depending on their adjacent tie condition

Groups	**Loss of support**	**Final Scores**															
		2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
F	**0%**	26	54	77	103	194	801	94	40	44	50	45	51	39	46	62	175
A	**16.67%**	5	12	21	27	78	242	25	15	14	11	14	10	16	17	18	59
B	**33%**	0	2	2	4	7	32	2	4	0	1	2	0	0	2	4	7
C	**46.44%**	0	0	1	0	3	12	1	0	0	0	0	0	0	1	0	0
A+B+C	**19.03%**	5	14	24	31	88	286	28	19	14	12	16	10	16	20	22	66

To model the behavior of ties with an initial score between 2.5 and 2.9, a weighted average of the loss of support was computed for Groups A+B+C (19.03%). In other words, the groups

Table 25 below represents ties with an initial score between 2.5 and 2.9 as well as their equivalent final scores depending on the adjacent tie condition (percent loss of support).

Table 25: Ties with initial score between 2.5 and 2.9

	Final Score															
Percent	**2.5**	**2.6**	**2.7**	**2.8**	**2.9**	**3**	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**	**3.6**	**3.7**	**3.8**	**3.9**	**4**

Loss of support																
0	1.4 %	2.8 %	4.1 %	5.4 %	10.2 %	42.1 %	4.9 %	2.1 %	2.3 %	2.6 %	2.4 %	2.7 %	2.1 %	2.4 %	3.3 %	9.2 %
16.67	0.9 %	2.1 %	3.6 %	4.6 %	13.4 %	41.4 %	4.3 %	2.6 %	2.4 %	1.9 %	2.4 %	1.7 %	2.7 %	2.9 %	3.1 %	10.1 %
33	0.0 %	2.9 %	2.9 %	5.8 %	10.1 %	46.4 %	2.9 %	5.8 %	0.0 %	1.4 %	2.9 %	0.0 %	0.0 %	2.9 %	5.8 %	10.1 %
46.44	0.0 %	0.0 %	5.6 %	0.0 %	16.7 %	66.7 %	5.6 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	0.0 %	5.6 %	0.0 %	0.0 %

In Table 25, the percentages represent the probability for a tie (with an initial score between 2.5 and 2.9) to get a specific Final Score, given the Percent loss of adjacent support (i.e., belonging to support Group F, A, B or C).

Plotting the values in Table 25 gives Figure 36 which represents the percentage of ties with an initial score between 2.5 and 2.9 (probability), their specific final score, as well as their respective loss of support.

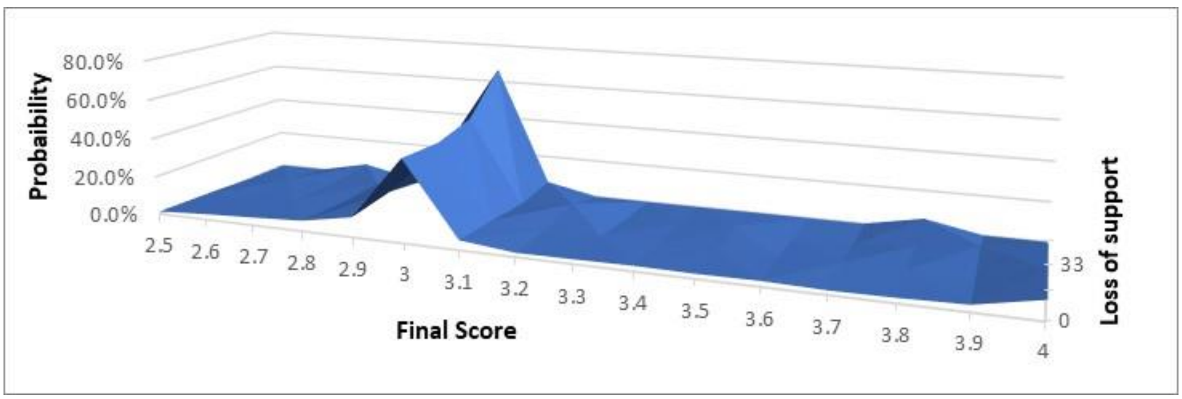


Figure 36: Probability of Final Scores as a function of Loss of support for ties with an initial score between 2.5 and 2.9

The loss of support for the Group A+B+C is computed using a weighted average of the three groups. Table 26 below represents the percentage of ties with an initial score between 2.5 and 2.9 and their final scores for Group F (with 0 percent loss of support) and Groups A+B+C (with 19% average loss of support).

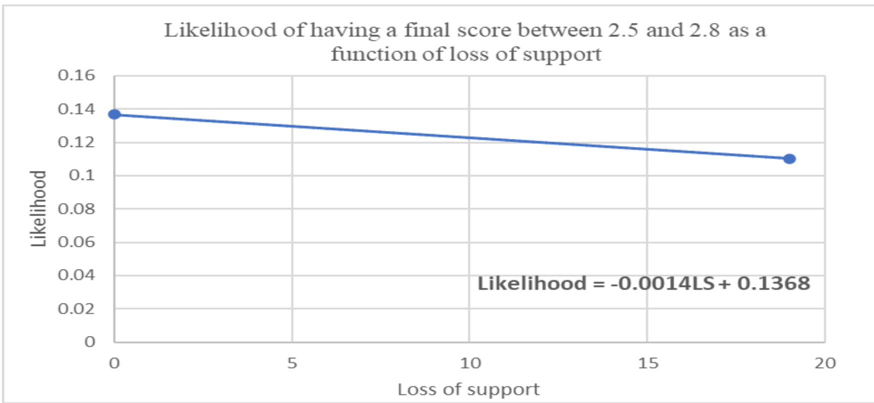
Table 26: Ties with initial score between 2.5 and 2.9

Groups	**Loss of support**	**Final Score**															
		2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
F	**0**	1.4 %	2.8 %	4.1 %	5.4 %	10.2 %	42.1 %	4.9 %	2.1 %	2.3 %	2.6 %	2.4 %	2.7 %	2.1 %	2.4 %	3.3 %	9.2 %
A+B +C	**19%**	0.7 %	2.1 %	3.6 %	4.6 %	13.1 %	42.6 %	4.2 %	2.8 %	2.1 %	1.8 %	2.4 %	1.5 %	2.4 %	3.0 %	3.3 %	9.8 %
F	**0**	13.7%				64.3%						22.0%					
A+B +C	**19%**	11.0%				66.6%						22.4%					

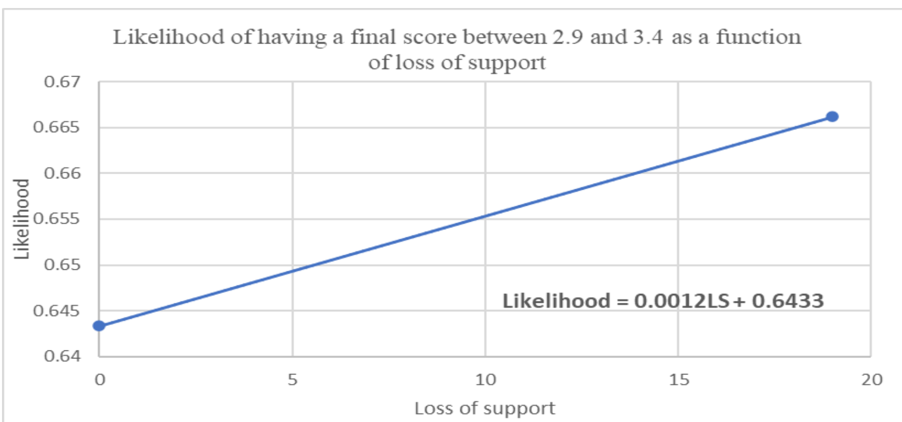
The analysis was conducted for the following ranges:

- For Initial Score between 2.5 and 2.9 and Final Scores between 2.5 and 2.8: **Equation D1**
- For Initial Score between 2.5 and 2.9 and Final Scores between 2.9 and 3.4: **Equation D2**
- For Initial Score between 2.5 and 2.9 and Final Scores between 3.5 and 4: **Equation D3**

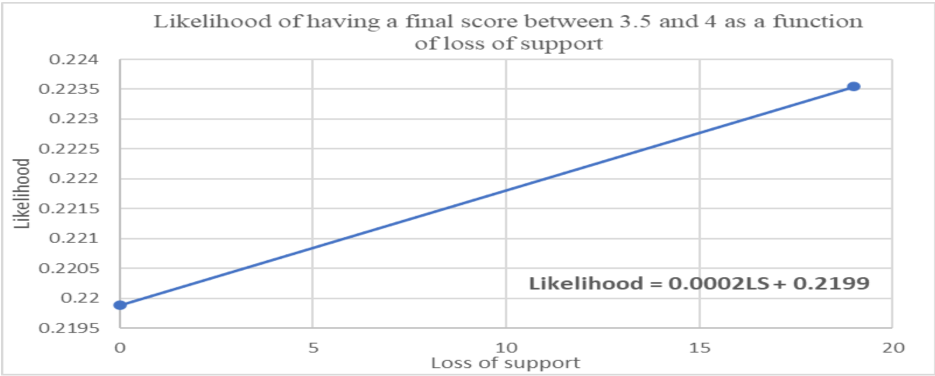
Figure 37 A, B, and C show the regression function representing the probability of having a final score between 2.5 and 2.8 (Equation D1), 2.9 and 3.48 (Equation D2), and 3.5 and 4 8 (Equation D3) respectively. Note; in Figure 37, Likelihood refers to the probability.



A: Probability function of having a final score 2.5 to 2.8



B: Likelihood function of having a final score 2.9 to 3.4



C: Likelihood function of having a final score 3.5 to 4

Figure 37: Likelihood function of having a final score between 2.5 and 2.8, 2.9 and 3.4, and 3.5 and 4

From Figure 37, the resulting regression function represent the Probability equations (Equation D1, D2, and D3) and can be summarized as follow:

Equation D1	**Probability = -0.014 *LS + 0.1368**
Equation D2	**Probability = 0.0012 *LS + 0.6433**
Equation D3	**Probability = 0.0002*LS + 0.2199**

where LS is the loss of support.

6.1.5. Equation E: Initial Score between 3 and 3.4

In this section, an equation (Equation E) was generated to model the behavior of ties with an initial score between 3 and 3.4.

Table 27 represents the percentage of ties with an initial score between 3 and 3.4 and their final scores depending on their adjacent tie condition. The initial score represents the score in 2016, while the final score represents the score in 2019.

Table 27: Ties with initial score between 3 and 3.4

		Final Score	
Initial Score	Percent Loss of support	3-3.4	3.5-4
3-3.4	0	43.30%	56.70%
	16.67	46.50%	53.50%
	33	48.60%	51.40%
	46.44	38.90%	61.10%

Table 27 represents ties with an initial score between 3 and 3.4 and their final scores depending on their adjacent tie condition.

Using MATLAB, a surface fit was performed to model the final score, the percent loss of support and the percentage. An equivalent equation was generated.

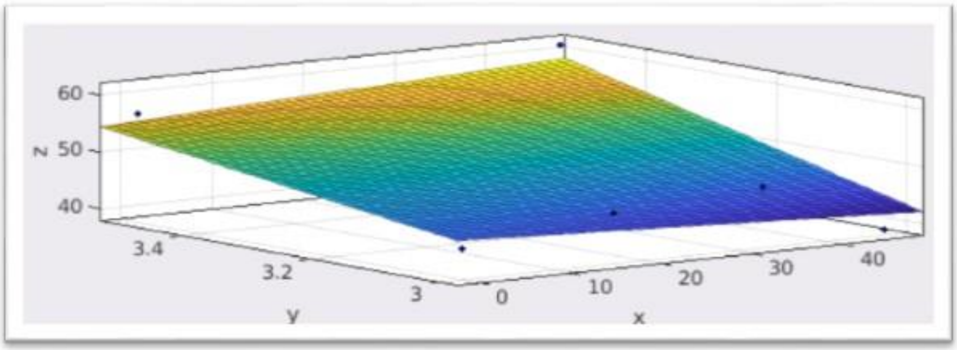


Figure 38: Surface fit for probability of a final score for an initial score between 3 and 3.4

Figure 38 represents the MATLAB surface fit output such that:

- x represents the percent loss of support,
- y represents the final score, and
- z represents the percentage of ties (in the dataset) having a final score of y and a loss of support of x and an initial score between (3 and 3.4).

The surface resulting equation f(x, y) (Equation E), represents the probability that a tie with initial score (between 3 and 3.4) and a loss of support of x gets a final score of y in 3 years as follows:

Equation E $$f(x,y) = p00 + p10*x + p01*y + p11*x*y$$

The Coefficients (with 95% confidence bounds) are:

p00 =	-4.584
p10 =	-0.7987
p01 =	16.8
p11 =	0.2458

Goodness of fit parameters can be summarized by:

- SSE: 97.81
- R-square: 0.7318
- Adjusted R-square: 0.3741
- RMSE: 5.71

6.1.6. Summary

In this section, different surface fittings were performed in order to get an appropriate equation describing the behavior of the ties, depending on the initial tie scores as well as the loss of adjacent support. No one equation properly modelled the full range of data so a series of equations were developed. These different equations describe and predict the probability of a final score Y in 3 years given the loss of support X for ties having a specific initial score, for each defined range. The equations considered the loss of support as a variable and the surface fittings were performed for each range of initial score separately:

- Initial Tie Scores between 1 and 1.4, resulting in Equation A
- Initial Tie Scores between 1.5 and 1.9, resulting in Equations B1, B2, and B3
- Initial Tie Scores between 2 and 2.4, resulting in Equation C
- Initial Tie Scores between 2.5 and 2.9, resulting in Equation D1, D2, and D3
- and Initial Tie Scores between 3 and 3.4, resulting in Equation E.

The summary of the modeling equation is shown in the following section.

6.2. Summary of Equations

The equations describing the probability for a tie with initial score SI and loss of support Ls to have a final score SF can be summarized in Table 28.

Table 28: Summary of Equations

Initial Score (SI)		Final Score (SF)		Equation P(SI, Ls, SF)	p00	p10	p01	p11	p02	p12	p03
1	**1.5**	1	4	**P(SI,Ls, SF)** = p00 + p10*Ls + p01*SF + p11*Ls*SF + p02*SF^2	94.91	-0.18	-55.21	0.08	7.94	0.00	0.00
1.5	**2**	1	2.9	**P(SI,Ls, SF)** = p00 + p10*Ls + p01*SF + p11*Ls*SF + p02*SF^2 + p12*Ls*SF^2 + p03*SF^3	-346.8	-0.81	535.4	0.66	-233.50	-0.12	31.18
		3	3.5	**P(SI,Ls, SF)** = p00 + p10*Ls + p01*SF + p11*Ls*SF + p02*SF^2	57.25	-0.11	-8.01	0.04	-2.55	0.00	0.00
		3.6	4	**P(SI,Ls, SF)** = p00 + p10*Ls + p01*SF + p11*Ls*SF + p02*SF^2 + p12*Ls*SF^2 + p03*SF^3	-346.8	-0.81	535.4	0.66	-233.50	-0.12	31.18
2	**2.5**	2	4	**P(SI,Ls, SF)** = p00 + p10*Ls + p01*SF + p11*Ls*SF + p02*SF^2	-67.96	-0.44	89.93	0.16	-19.60	0.00	0.00
2.5	**3**	2.5	2.8	**P(SI,Ls, SF)** = p00 + p10*Ls	-0.0014	0.14	0.00	0.00	0.00	0.00	0.00
		2.9	3.4	**P(SI,Ls, SF)** = p00 + p10*Ls	0.0012	0.64	0.00	0.00	0.00	0.00	0.00
		3.5	4	**P(SI,Ls, SF)** = p00 + p10*Ls	0.0002	0.22	0.00	0.00	0.00	0.00	0.00
3	**3.4**	3	4	**P(SI,Ls, SF)** = p00 + p10*Ls + p01*SF + p11*Ls*SF	-4.584	-0.7987	16.8	0.2458	0.00	0.00	0.00

7. Introducing the Time Variable to the Probability of Tie Degradation

After modeling the tie score changes that happen within 3 years, the time variable was introduced to the calculated probabilities, to allow for determination of rate of tie degradation. In this analysis, the increase in the tie degradation likelihood with time, is used to calculate a rate of degradation which, in turn, can be used to calculate tie life.

Both exponential degradation and linear degradation of wood ties are examined and presented in this section.

The objective of this study is being able to predict the amount of time it will take for a "good tie" to have a high probability of failure based on its adjacent tie condition (loss of adjacent tie support). Failure is defined using a probability threshold; for example, 75%.

The previous models allowed for the determination of the probability of a tie changing condition (from an initial score SI to a final score SF) in 3 years. The following section expands upon this and extends the probability equation to include the time variable.

Introduction the time variable to the previous results is done in two different ways:

- Exponential crosstie degradation over time
- Linear crosstie degradation over time

The inputs are:

- Threshold probability; set here to be 75% as a default case.
- Initial Score
- Final Score
- Average tie life; taken from external studies and dependent on numerous factors to include type of wood, weather/environment, traffic type and density, etc.

The output is:

- The required time for the probability (of tie failure) of a tie moving from an initial score SI to a final score SF to be higher than the set threshold (75% default case).

In order to model the time (t) for the probability to be higher than Tr (a threshold input), for a tie with initial condition score (SI) and loss of support (LS) to reach a final score (SF), the following abbreviations presented in Table 29, will be used:

Table 29: Used Abbreviations

Ls	Loss of Support
SF	Final Score
SI	Initial Score
P(SI,LS, SF)	Probability that a tie with initial score (SI) and a loss of support of LS gets a final score of (SF) in 3 years (functions from surface modeling)
T	An average tie life (based on wood type, historic data, weather,….) should be an input

Tr2	The required time for the probability P(SI,LS,SF) to be higher than threshold (Tr2)
Tr1	Probability Threshold (default case is 75%)

- *Inputs:*
  - Initial Score SI
  - Loss of Support LS
  - A potential final score: SF; as the objective is to determine what the probability to get this final score SF is.
  - An average tie life (T): based on traffic, wood type, historic data, weather…
  - A threshold of probability (default case is 75%)
- *Outputs:*
  - Change of probability over time to reach a final score (inputted as SF)
  - Time for the probability of reaching initial condition score SI (inputted) to be higher than the defined threshold (75%).

7.1. Exponential Degradation of Wood Ties

In this analysis, it is assumed that the degradation of ties is exponential, as shown in Figure 39, and the tie score follows an exponential growth trend over time[7]. This will be the basis for the tie life and probability growth modelling.

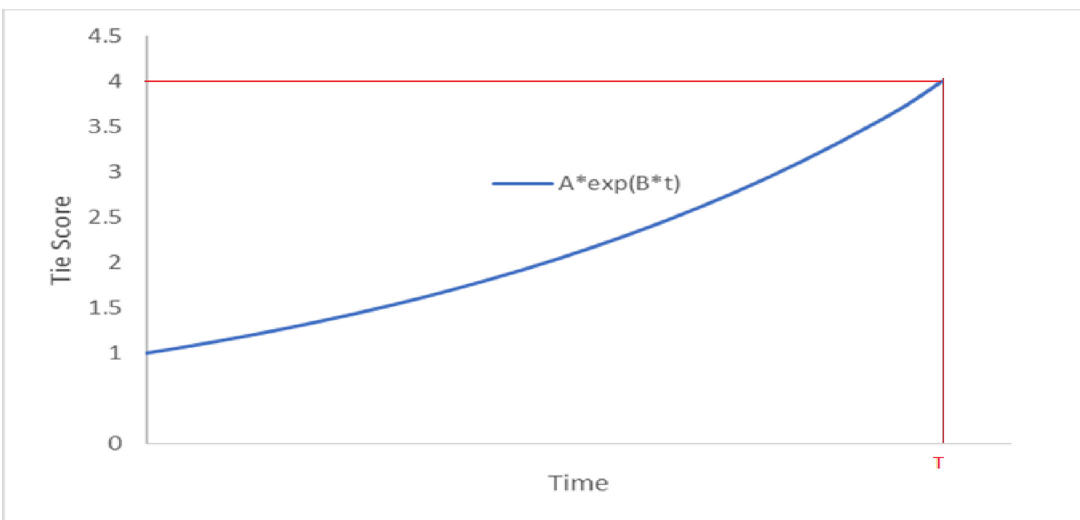


Figure 39: Exponential tie degradation

In Figure 39 above, T (on the x-axis representing Time) represents the average tie life (which is an input and depends on many other important factors as discussed in the previous sections), or the amount of time it takes for the tie score to reach 4.

[7] This statement can be found in an Aurora presentation in : http://railtec.illinois.edu/wp/wp-content/uploads/pdf-archive/8.6_Euston.pdf

It is to be noted that, in this analysis, the tie life is defined to be the time the tie condition score goes from 1 to 4. Note that the mathematical proof can be found in Appendix B.

The tie condition or Tie Score can be modeled by Equation 3:

$$\text{Tie Score} = 4^{\left(t/_{T(LS)}\right)} \qquad \text{Equation 3}$$

where:

- T (LS) = (1.444 LS^2 - 1.322 LS + 0.9931) * T (from Equation 1)
- And t is the time in years.

7.1.1. ΔT: Time to go from a score SI (initial) to a score SF (final)

Using Equation 3, the time to go from a score SI (initial) to a score SF (final) is

$$\Delta T = \frac{(1.444\, Ls2\; - \;1.322\, Ls\; + \;0.9931) * T}{\ln(4)} * \ln(\frac{SF}{SI}) \qquad \text{Equation 4}$$

Note that the mathematical proof can be found in Appendix B.

7.1.2. Exponential Increase of Probability

Based on the data and the modeling completed in the previous section, the probability to go from score SI to Score SF in 3 years is determined based on the loss of support, and will be referred to hereinafter as P(SI, LS, SF), where SI is the initial Score, LS the loss of support and SF the final score.

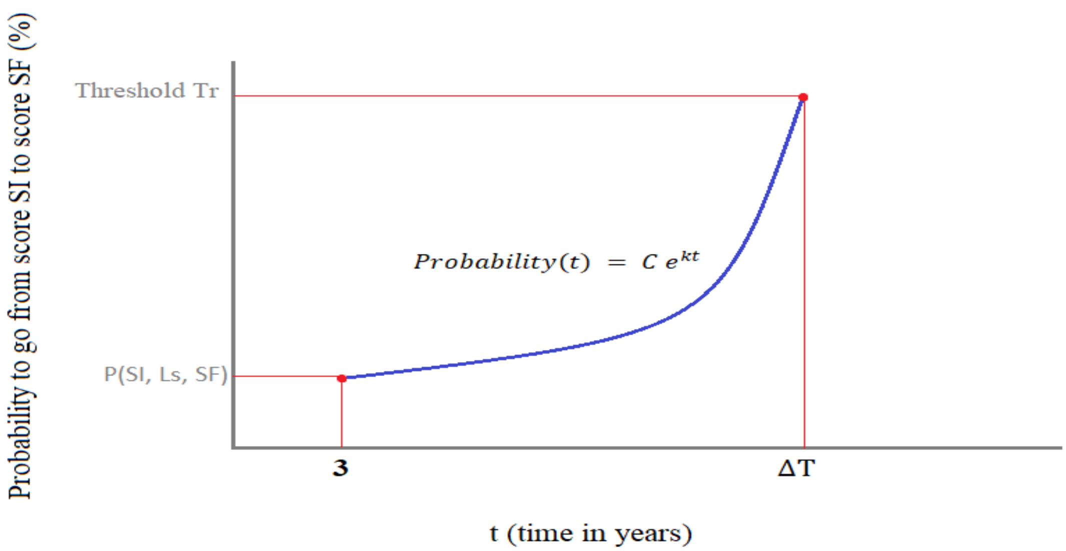


Figure 40: Exponential growth of probability over time

In Figure 40, P (SI, LS, SF) is computed using the equations from the surface fitting. It represents the probability that a tie would go from Score SI to Score SF within 3 years based on the loss of adjacent support.

Assuming that the probability is exponentially increasing over time, the change of probability over time can be described by Equation 5. Note that the mathematical proof can be found in Appendix B.

$$\mathrm{Probability(t)} \;=\; (\mathrm{P(SI, Ls, SF)})^{(\frac{\Delta T}{\Delta \mathrm{T}-3})} * \;(\mathrm{Tr1})^{(\frac{3}{3-\Delta \mathrm{T}})}\; \mathrm{e}^{\frac{\ln(\frac{\mathrm{Tr1}}{\mathrm{P(SI,Ls,SF)}})}{\Delta \mathrm{T}-3}\mathrm{t}} \qquad \text{Equation 5}$$

7.1.3. Determining t(Tr1): the time it takes for the probability to be higher than threshold Tr1

Using Equation 4, Equation 5, and the mathematical proof in Appendix B, the time it takes for the probability of a tie to move from score SI to a final Score SF to be higher than threshold Tr1 is expressed by Equation 6:

$$\mathbf{t(Tr)} = \left(\frac{(\mathbf{1.444}\,\boldsymbol{Ls}\mathbf{2} \;-\; \mathbf{1.322}\,\boldsymbol{Ls} \;+\; \mathbf{0.9931}) * \boldsymbol{T}}{\mathbf{ln(4)}} * \mathbf{ln}\left(\frac{\boldsymbol{SF}}{\boldsymbol{SI}}\right) - \mathbf{3}\right) * \frac{\mathbf{ln}\left(\dfrac{\mathbf{Tr1}\,\hat{}\,\left(\frac{0.5}{\mathrm{SF-SI-0.5}} * \frac{3}{\frac{(1.444\,Ls2\,-\,1.322\,Ls\,+\,0.9931)*T}{\ln(4)} * \ln(\frac{SF}{SI}) - 3}\right)}{\mathbf{P(SI, Ls, SF)}\,\hat{}\,\left(\frac{\frac{(1.444\,Ls2\,-\,1.322\,Ls\,+\,0.9931)*T}{\ln(4)} * \ln(\frac{SF}{SI})}{\frac{(1.444\,Ls2\,-\,1.322\,Ls\,+\,0.9931)*T}{\ln(4)} * \ln(\frac{SF}{SI}) - 3}\right)}\right)}{\mathbf{ln}\left(\dfrac{\mathbf{Tr1}}{\mathbf{P(SI, Ls, SF)}}\right)} \qquad \text{Equation 6}$$

where:

- LS: Loss of support
- Tr1: The threshold set
- T: Average tie life
- SI: Initial Score
- SF: Final Score
- P(SI, LS, SF): the probability for a tie with an initial Score SI to reach a final score SF in 3 years, from the modelling presented in the previous section.

7.2. Linear Degradation of Wood Ties

In this section, the degradation of tie condition was assumed to be linear, with the tie degradation score following a linear relationship over time.

This assumed linear behavior statement was used for the tie life and probability growth modelling.

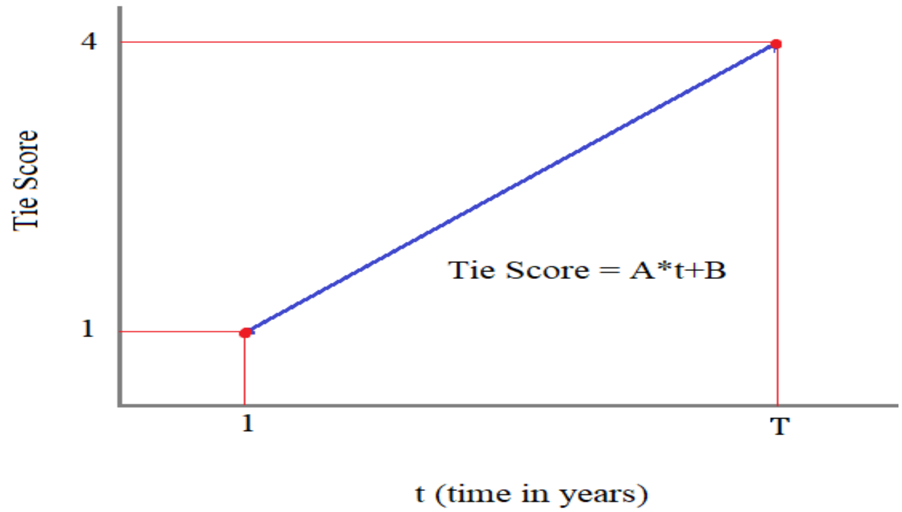


Figure 41: Linear tie degradation

In Figure 41, T (on the x-axis representing time in years) represents the average tie life (which is an input, and depends on many important factors), or the amount of time it takes for the tie condition score to go from 1 to 4.

The Tie Score (tie condition) is then modelled by Equation 7. Note that the mathematical proof can be found in Appendix C.

$$\text{Tie Score (t)} = 1 + \frac{3t - 1}{T(Ls) - 1} \qquad \text{Equation 7}$$

where,

T (LS) = (1.444 LS^2 - 1.322 LS + 0.9931) * T,

T is the input average tie life, and t is the time in years.

7.2.1. ΔT: Time to go from a score SI (initial) to a score SF (final)

From Equation 7,

$$\text{Tie Score SI} = 1 + \frac{3 * tSI - 1}{T(Ls) - 1}$$

$$\text{Tie Score SF} = 1 + \frac{3 * tSF - 1}{T(Ls) - 1}$$

Such that tSI and tSF represent respectively the time for a tie to reach score SI and Score SF, respectively. And T(LS) is the average tie life (based on the loss of support).

$$tSI = \frac{SI * (T(Ls) - 1) + 4 - T}{3}$$
$$tSF = \frac{SF * (T(Ls) - 1) + 4 - T}{3}$$

So, the time to go from a score SI (initial) to a score SF (final) is described by Equation 8.

$$\Delta T = tSF - tSI = \frac{S(F - FI) * (T(LS) - 1)}{3} \quad \text{Equation 8}$$

7.2.2. Linear Growth of Probability (Over Time)

Based on the data and the modeling completed in the previous section, the probability to go from score SI to Score SF in 3 years is determined based on the loss of support, and will be referred to hereinafter as P(SI, LS, SF) where SI is the initial Score, LS the loss of support and SF the final score.

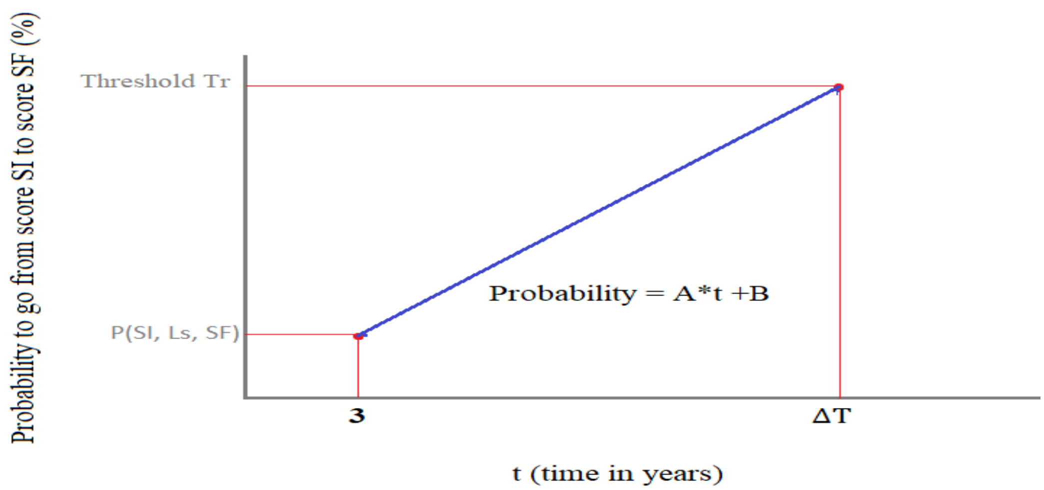


Figure 42: Linear growth of probability over time

Assuming that the probability is linearly growing over time, as shown in Figure 42, the change of probability over time can be described by Equation 9. The mathematical proof can be found in Appendix C.

$$\text{Probability}(t) = \frac{P(SI, Ls, SF) - Tr}{3 - \Delta T} * t + \frac{3 * Tr - \Delta T * P(SI, Ls, SF)}{3 - \Delta T} \quad \text{Equation 9}$$

Similar to the previous section, using Equation 8, Equation 9, and the mathematical proof in Appendix C, the time it takes for the probability of a tie to move from score SI to a final Score SF to be higher than threshold Tr is expressed by the Equation 10.

$$t = \frac{\frac{S(F - FI) * ((1.444\ Ls2\ -\ 1.322\ Ls\ +\ 0.9931) * T - 1)}{3} * \left(P(SI, Ls, SF) - Tr^{\left(\frac{0.5}{SF-SI-0.5}\right)}\right) + 3 * \left(Tr^{\left(\frac{0.5}{SF-SI-0.5}\right)} - Tr\right)}{P(SI, Ls, SF) - Tr} \quad \text{Equation 10}$$

where:

- LS: Loss of support
- Tr: The threshold set
- T: Average tie life
- SI: Initial Score
- SF: Final Score
- P(SI, LS, SF): the probability for a tie with an initial Score SI to reach a final score SF in 3 years, from the modelling presented in the previous section.

7.3. Summary

In this chapter, the time variable was introduced to the calculated tie degradation probabilities. The change in failure probability over time was considered based on the tie behavior modelled in chapter 9.

The introduction of the time variable to the previous results was performed in two different ways:

- Exponential crosstie degradation over time
- Linear crosstie degradation over time

The resulting equations allow for the prediction of the amount of time it will take for a “good tie” to fail or have a high probability of failure (based on a set threshold such as the default value of 75%) based on its adjacent tie condition (loss of adjacent tie support).

8. Tie Life Reconstruction

In track, wood crossties do not fail simultaneously even when installed at the same time [11]. This is due to variations in maintenance, tie replacement, and the normal statistical distribution of tie degradation (related to the anisotropic properties of wood). The distribution of failures represents a skewed normal distribution around an average tie life in track [14]. Since the tie condition data in the data set represents different tie conditions, which in turn are representative of different periods in the tie life, this condition data can be used to create a piecewise reconstruction of an average tie life.

As noted, the tie condition is determined by the Aurora automated inspection system which provides a tie condition score to each cross-tie. Each tie is individually scanned, and analyzed to assess its condition, which is based on more than 20 different variables, including plate cut, splitting, internal decay, etc. The system then assigns a grade to each tie on a scale of 1 to 4 describing its condition, 1 being the best and 4 being failed [19].

Using this scoring system, the tie life reconstruction is performed such that a score of 1 is equivalent to the initial year in the tie life with the tie considered "new", and a score of 4 is equivalent to the end of the tie life, with the tie considered failed.

Since the assigned tie scores are decimal, it is possible to monitor the tie condition over the inspection interval and use that rate of change of condition to predict forward the tie life. As noted, this required a very accurate alignment process [9] using the 96,421 ties from both 2016 and 2019 inspection years together with their corresponding condition values.

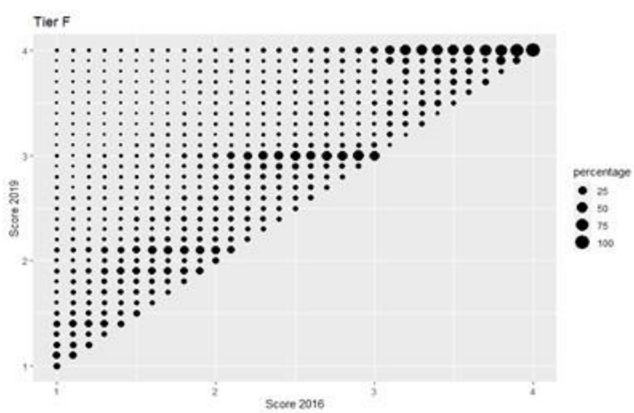
8.1. Regression Function

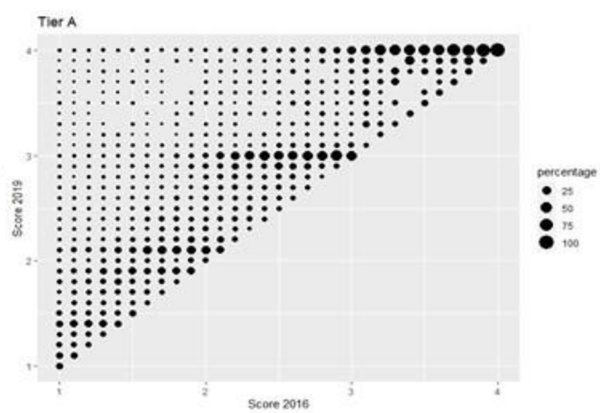
The first method used to reconstruct an average tie life was regression. Regression functions were developed based on the distributions of the different tie score transitions from 2016 to 2019 in different support groups and tiers (F, F+A, A+B+C, B+C). These functions were then used recursively to predict the tie score in year t+3 knowing the score in t.

$$\text{Score}\ (t+3) = f\ (\text{Score}(t)),$$

such that f is the resulting regression function, and t is time in years. This way, a piecewise reconstruction of the average tie life was performed and enabled to compare the tie degradations rates with respect to loss of adjacent support.

Plotting all the tie score transitions from Figures 28-31 gives Figure 43 which represents the different tie scores in 2016 with their equivalent changed tie score in 2019, as well as the percent of ties having each transition.





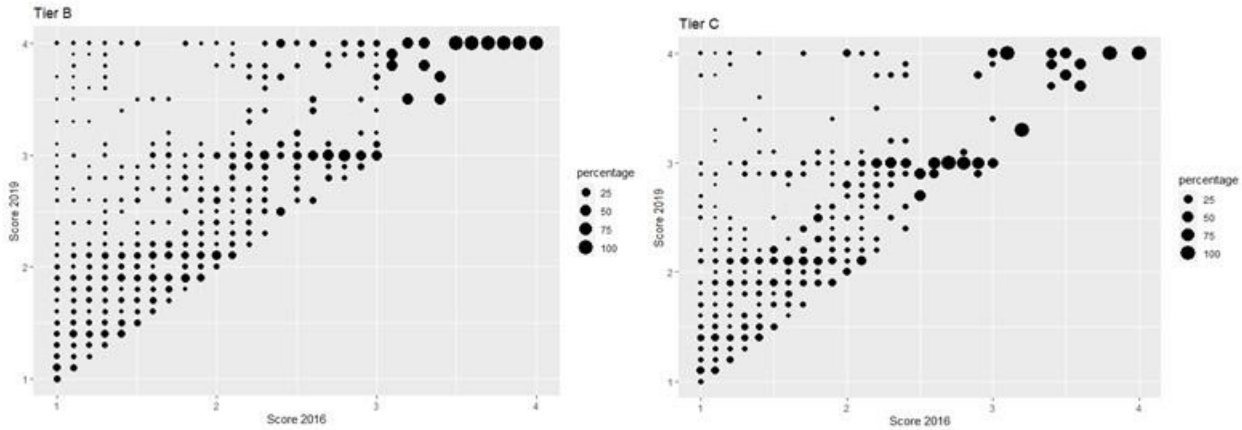


Figure 43: Percentages of tie score transitions for Groups A, B, C, and F

The tie score transitions that had a percentage higher than 10%, were kept for this study, as they represent significant condition changes.

It should be noted that, because the groups were unbalanced, the tie life reconstruction was performed on different group bundles, having different weighted[8] average loss of adjacent support values.

- Group F (loss of support= 0) is to be compared to Tier A+B+C (having an average loss of support of 19%)
- Tier A+F (having an average loss of support of 3%) is to be compared to Tier B+C (having an average loss of support of 37%)

8.1.1. Tie Life Reconstruction: Group F

A regression analysis of ties in group F (all adjacent ties in good condition) was performed and is presented in Figure 44. This figure represents tie scores in 2016 with their equivalent tie score transitions in 2019 in Group F. Note, only tie score transitions with a percentage higher than 10% of the ties were used in Figure 44.

[8] Weighted based on number of ties in each group

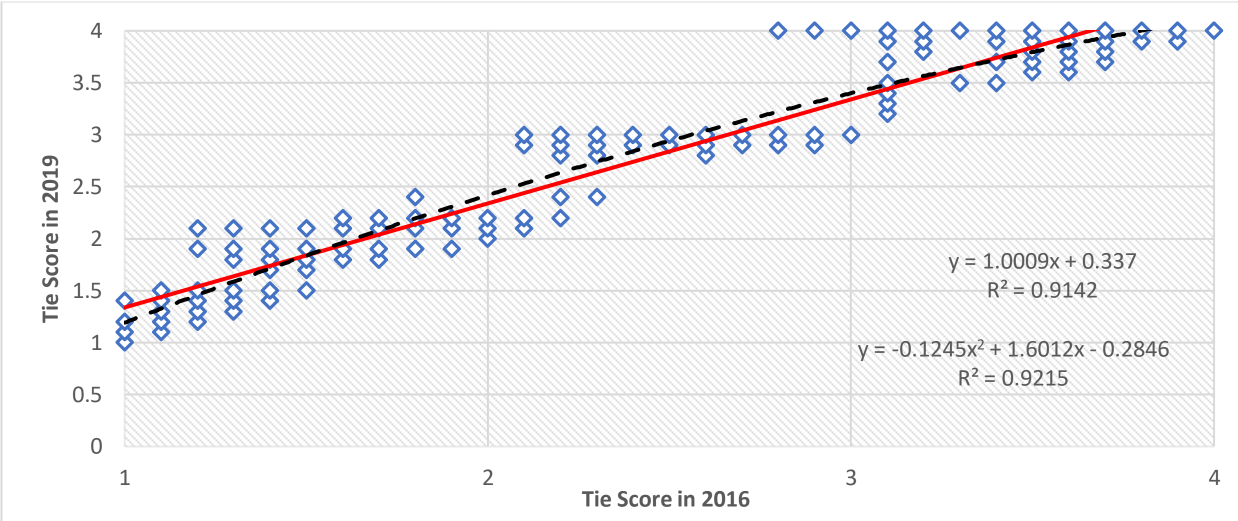


Figure 44: Tie Score 2016 Vs Tie Score 2019 for Group F

In order to describe the behavior of the data, two regression lines were fitted to the data with an R^2 of 0.91 and 0.92 respectively:

- Linear regression, with the resulting equation: $y= 1.0009x+0.337$
- Quadratic regression, with the resulting equation: $y= -0.1245x^2 +1.6012x-0.2846$

where x represents tie condition scores in 2016, while y represents tie condition scores in 2019. The equations can thus be described as:

Linear regression: Score (2019) = 1.0009* Score (2016) +0.337
Quadratic regression: Score (2019) = - 0.1245 Score $(2016)^2$ +1.6012* Score (2016)-0.2846

Because the tie score transitions were recorded within an interval of 3 years (between 2016 and 2019), the equations can be generalized as follow:

Linear regression: Score (t+3) = 1.0009* Score (t) +0.337
Quadratic regression: Score (t+3) = - 0.1245 Score $(t)^2$ +1.6012* Score (t)-0.2846

where Score(t) is the tie score at time t (in years) and Score (t+3) is the tie score at time t+3 (in years).

Assuming a tie score of 1 will occur in year 1, and using the generalized equations above, the tie life was reconstructed for group F and the results presented in Table 30.

Table 30: Tie life reconstruction results for group F

Linear Regression	
Time (years)	Tie Score
1	1

Quadratic Regression	
Time (years)	Tie Score
1	1

4	1.3		4	1.2
7	1.7		7	1.5
10	2.0		10	1.8
13	2.3		13	2.2
16	2.6		16	2.6
19	2.9		19	3.0
22	3.2		22	3.4
25	3.5		25	3.7
28	3.8		28	3.9
31	4.14		31	4.06

Using the results of the linear regression function, the average time it takes a tie to reach score of 3.8 is 28 years. Using the quadratic regression function, the average time it takes a tie to reach score of 3.9 is 28 years.

Interpolating the results in **Error! Reference source not found.** 30, the average time it takes a tie to reach score of 4 is:

- For the linear regression: 29.5 years
- For the quadratic regression: 29.8 years

8.1.2. Tie Life Reconstruction: Tier A+B+ C

A regression analysis of ties of combined Groups A, B, and C which is defined as Tier A+B+C (at least one adjacent tie in poor condition) was performed and is presented in Figure 45. This figure represents tie scores in 2016 with their equivalent tie score transitions in 2019 in Tier A+B+C. Note, only tie score transitions with a percentage higher than 10% of the ties were used in Figure 45 below.

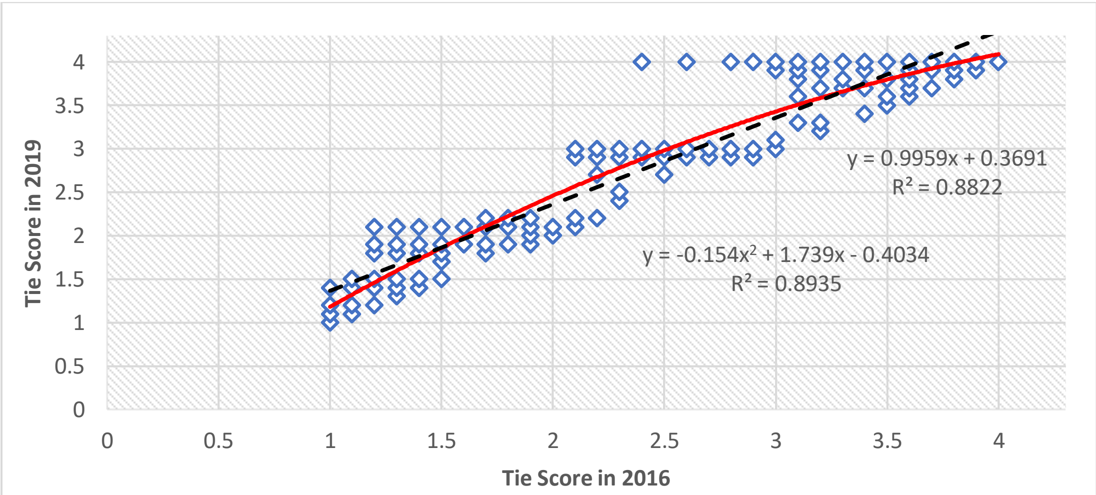


Figure 45: Tie Score 2016 Vs Tie Score 2019 for Tier A+B+C

In order to describe the behavior of the data, two regression lines were fitted to the data with an R^2 of 0.88 and 0.98 respectively:

- Linear regression with the resulting equation: $y = 0.9959\,x + 0.3691$
- Quadratic regression with the resulting equation: $y = -0.154x^2 + 1.739x - 0.4034$

where x represents tie condition scores in 2016, while y represents tie condition scores in 2019, so the equations can be described as:

Linear regression: Score (2019) = 0.9959 *Score (2016) + 0.3691
Quadratic regression: Score (2019) = - 0.154* Score (2016) ^{2} + 1.739* Score (2016) - 0.4034

Because the tie score transitions were recorded within an interval of 3 years (between 2016 and 2019), the equations can be generalized as follow:

Linear regression: Score (t+3) = 0.9959 *Score (t) + 0.3691
Quadratic regression: Score (t+3) = - 0.154* Score (t) ^{2} + 1.739* Score (t) - 0.4034

where Score(t) is the tie score at time t (in years) and Score (t+3) is the tie score at time t+3 (in years).

Assuming a tie score of 1 will occur in year 1, and using the generalized equations above, the tie life was reconstructed for Tier A+B+C and the results can be seen in Table 31.

Table 31: Tie life reconstruction results for Tier A+B+C

Linear Regression		Quadratic Regression	
Time (years)	Tie Score	Time (years)	Tie Score
1	1	1	1
4	1.4	4	1.2
7	1.8	7	1.5
10	2.2	10	1.9
13	2.6	13	2.2
16	3.0	16	2.7
19	3.4	19	3.2
22	3.8	22	3.6
25	4.15	25	3.9
		28	4.04

Using the results of the linear regression function, the average time it takes a tie to reach score of 3.8 is 22 years. Using the quadratic regression function, the average time it takes a tie to reach score of 3.9 is 25 years.

Interpolating the results in Table 31, the average time it takes a tie to reach score of 4 is:

- For the linear regression: 23.7 years

- For the quadratic regression: 27 years

8.1.3. Tie Life Reconstruction: Tier F+A

A regression analysis of ties of combined Groups F and A which is defined as Tier F+A was performed and is presented in Figure 46. This figure represents tie scores in 2016 with their equivalent tie score transitions in 2019 in Tier F+A.

Note; only tie score transitions with a percentage higher than 10% of the ties were used in Figure 46 below.

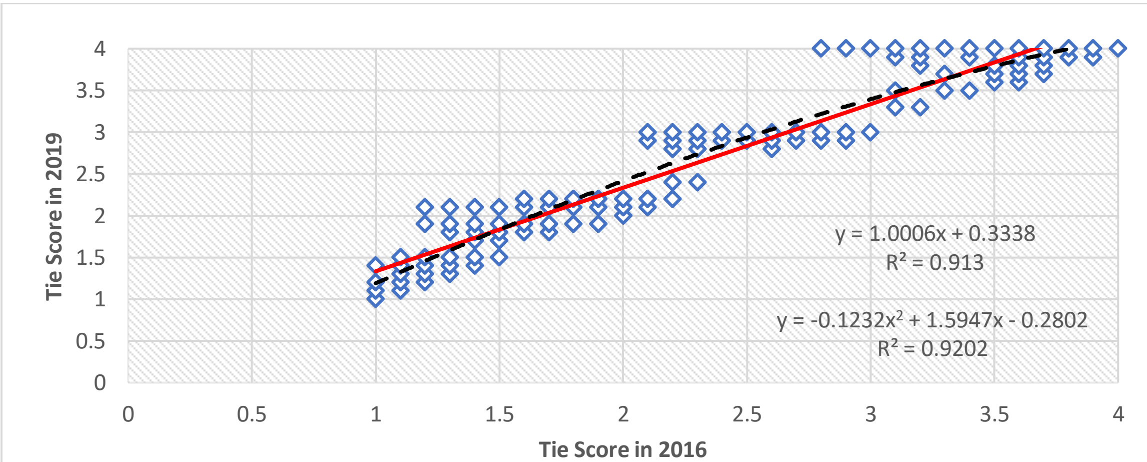


Figure 46: Tie Score 2016 Vs Tie Score 2019 for Tier F+A

In order to describe the behavior of the data, two regression lines were fitted to the data with an R^2 of 0.91 and 0.92 respectively:

- Linear regression, with the resulting equation: $y = 1.0006x + 0.3338$
- Quadratic regression, with the resulting equation: $y = -0.1232x^2 + 1.5947x - 0.2802$

where, x represents tie scores in 2016, while y represents tie scores in 2019, so the equations can be described as:

Linear regression: Score (2019) = 1.0006 * Score (2016) + 0.3338
Quadratic regression: Score (2019) = -0.1232* Score $(2016)^2$ + 1.5947* Score (2016) - 0.2802

Because the tie score transitions were recorded within an interval of 3 years (between 2016 and 2019), the equations can be generalized as follow:

Linear regression: Score (t+3) = 1.0006 * Score (t) + 0.3338
Quadratic regression: Score (t+3) = -0.1232* Score $(t)^2$ + 1.5947* Score (t) - 0.2802

where Score(t) is the tie score at time t (in years) and Score (t+3) is the tie score at time t+3 (in years).

Assuming a tie score of 1 will occur in year 1, and using the generalized equations above, the tie life was reconstructed for Tier F+A and the results can be seen in Table 32.

Table 32: Tie life reconstruction results for Tier F+A

Linear Regression	
Time (years)	Tie Score
1	1
4	1.3
7	1.6
10	1.9
13	2.2
16	2.5
19	2.8
22	3.1
25	3.4
28	3.7
31	4.04

Quadratic Regression	
Time (years)	Tie Score
1	1
4	1.2
7	1.5
10	1.8
13	2.2
16	2.6
19	3.0
22	3.4
25	3.6
28	3.9
31	4.07

Using the linear regression resulting function, the average time it takes a tie to reach score of 3.7 is 28 years. Using the quadratic regression resulting function, the average time it takes a tie to reach score of 3.9 is 28 years.

Interpolating the results in Table 32, the average time it takes a tie to reach score of 4 is:

- For the linear regression: 30.6 years
- For the quadratic regression: 29.7 years

8.1.4. Tie Life Reconstruction : Tier B+C

A regression analysis of ties of combined Groups B, and C with is defined as Tier B+C was performed and is presented in Figure 47. This figure represents tie scores in 2016 with their equivalent tie score transitions in 2019 in Tier B+C. Note, only tie score transitions with a percentage higher than 10% of the ties were used in Figure 47.

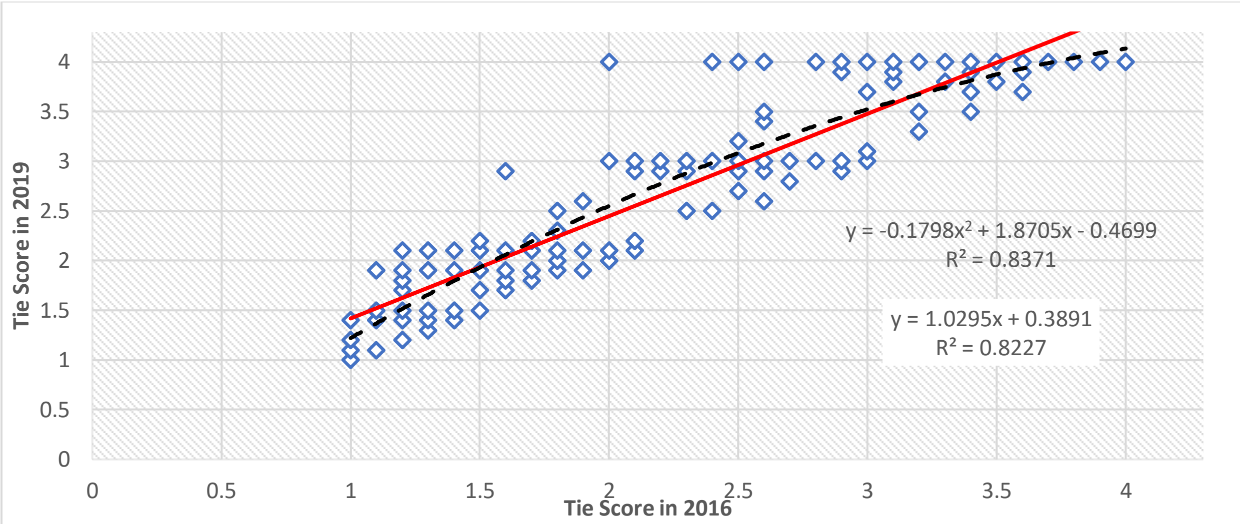


Figure 47: Tie Score 2016 Vs Tie Score 2019 for Tier B+C

In order to describe the behavior of the data, two regression lines were fitted to the data with an R^2 of 0.82 and 0.84 respectively:

- Linear regression, with the resulting equation: y = 1.0295x + 0.3891
- Quadratic regression, with the resulting equation: $y = -0.1798x^2 + 1.8705x - 0.4699$

where x represents tie scores in 2016, while y represents tie scores in 2019, so the equations can be described as:

Linear regression: Score (2019) = 1.0295 *Score (2016) + 0.3891
Quadratic regression: Score (2019) = - 0.1798 * Score $(2016)^2$ + 1.8705* Score (2016) - 0.4699

Because the tie score transitions were recorded within an interval of 3 years (between 2016 and 2019), the equations can be generalized as follow:

Linear regression: Score (t+3) = 1.0295 *Score (t) + 0.3891
Quadratic regression: Score (t+3) = - 0.1798 * Score $(t)^2$ + 1.8705* Score (t) - 0.4699

where Score(t) is the tie score at time t (in years) and Score (t+3) is the tie score at time t+3 (in years).

Assuming a tie score of 1 will occur in year 1, and using the generalized equations above, the tie life was reconstructed for Tier B+C and the results can be seen in Table 33.

Table 33: Tie life reconstruction results for Tier B+C

Linear Regression	
Time (years)	Tie Score

Quadratic Regression	
Time (years)	Tie Score

1	1		1	1
4	1.4		4	1.2
7	1.8		7	1.5
10	2.2		10	1.9
13	2.7		13	2.4
16	3.2		16	3.0
19	3.7		19	3.5
22	4.0		22	3.9
			23	4.09

Using the linear regression resulting function, the average time it takes a tie to reach score of 3.8 is 28 years. Using the quadratic regression resulting function, the average time it takes a tie to reach score of 3.9 is 28 years.

Interpolating the results in Table 33, the average time it takes a tie to reach score of 4 is:

- For the quadratic regression: 22.6 years.

8.2. Comparison

Observing the differences in equations and results for the different Groups/Tiers, to include both the linear and quadratic regression equations it is now possible to quantify this difference, and relate it to the loss of support. The comparison of the different tie life reconstructions is presented in this section.

8.2.1. Quadratic Regression Equation

Tables 30 through 33 presented the results of the tie life reconstruction for the noted tie Groups/Tiers. Focusing on the quadratic regression results, plots of tie condition score vs time are presented in Figure 48 for Group F and Tier A+B+C and in Figure 49 for Tier B+C and Tier A+F.

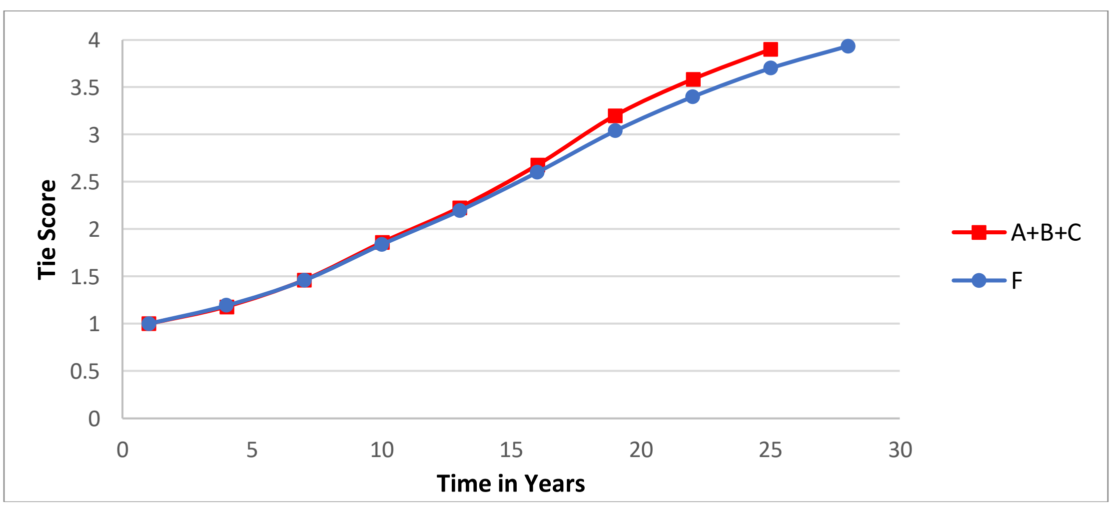


Figure 48: Time in years Vs Tie Score Using Quadratic function for Tier A+B+C and Group F

From Figure 48, it can be seen that group F and group A+B+C behave similarly until condition score 2.2 is reached. Then, Tier A+B+C shows a faster rate of degradation (and shorter tie life) than Group F. It takes a tie in Tier A+ B+ C on average 25 years to reach score 3.9, while it takes 28 years to reach the same score for a tie belonging to group F (0% loss of support). Thus, the ties with poorer adjacent tie support conditions show a faster rate of degradation and a shorter projected life than those with good adjacent tie support condition.

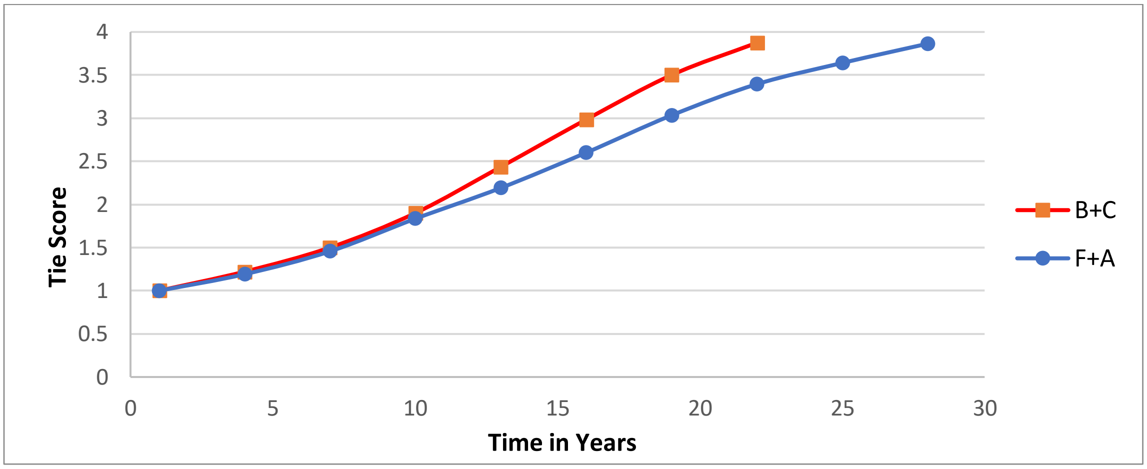


Figure 49: Time in years Vs Tie Score Using Quadratic function for Tier B+C and Tier F+A

From Figure 49, It can be seen that Tier F+A and Tier B+C behave similarly until condition score 1.8 is reached. Then, Tier B+C shows a faster rate of degradation (and shorter tie life) than Tier A+ F. It takes a tie in Tier B+ C on average 22 years to reach score 3.9(37% loss of support), while it takes 28 years to reach the same score for a tie belonging to Tier F+A (3% loss of support). So here too, the ties with poorer adjacent tie support conditions show a faster rate of degradation and a shorter projected life than those with good adjacent tie support condition.

8.2. 2. Linear Regression Resulting Equation

Likewise, examining the results of the linear regression equations from Tables 30 through 33, plots of tie condition score vs time are presented in Figure 50 for Group F and Tier A+B+C and in Figure 51 for Tier B+C and Tier A+F.

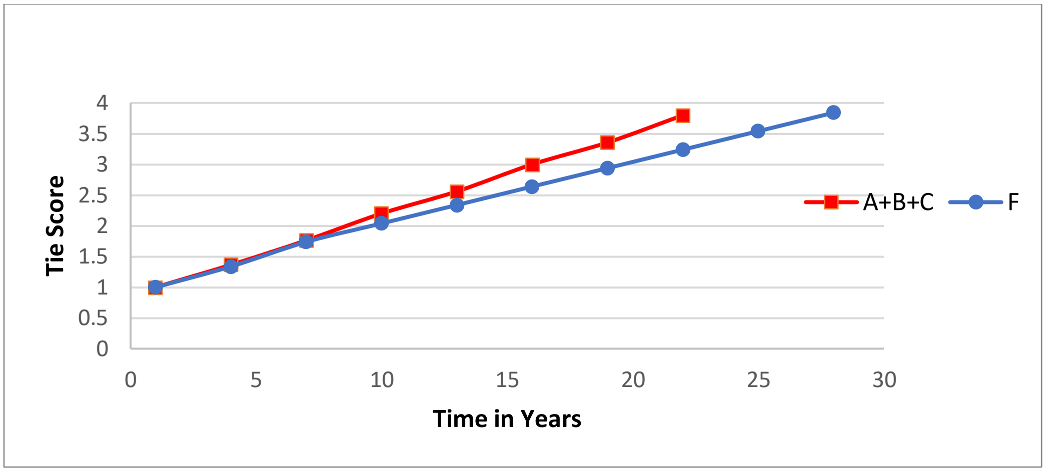


Figure 50: Time in years Vs Tie Score Using the Linear function for Tier A+B+C and Group F

From Figure 50, it can be seen that Group F and Tier A+B+C behave similarly until condition score 1.7 is reached. Then, Tier A+B+C shows a faster rate of degradation (and shorter tie life) than Group F. It takes a tie in Tier A+ B+ C on average 22 years to reach score 3.9 (19% loss of support), while it takes 28 years to reach the same score for a tie belonging to Group F (0% loss of support).

Thus, as was shown for the quadratic equations, these results show a faster rate of degradation and shorter tie life for the ties with poor adjacent ties (poor support) as compared to all good ties on either side.

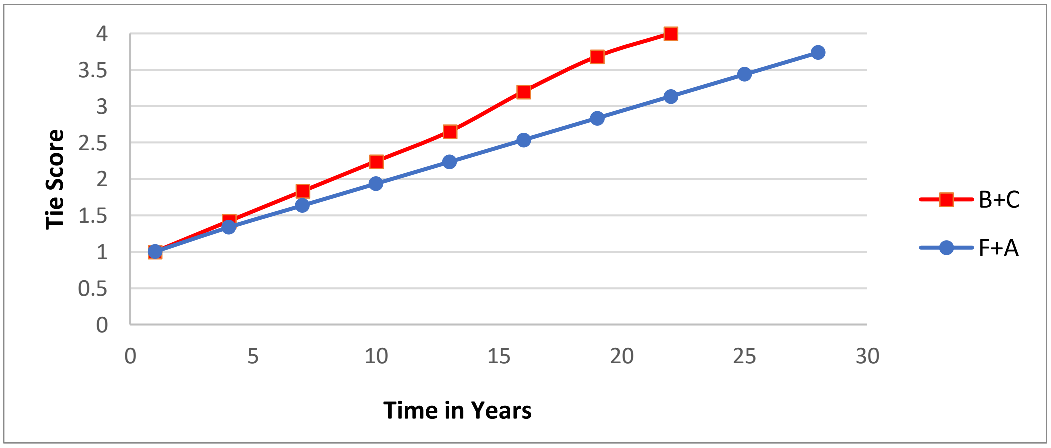


Figure 51: Time in years Vs Tie Score Using the Linear function for Tier B+C and Group F

From Figure 51, It can be seen that Tier F+A and Tier B+C behave similarly until condition score 1.5 is reached. Then, Tier B+C shows a faster rate of degradation (and shorter tie life) than Tier A+ F. It takes a tie in Tier B+ C on average 22 years to reach score 4 (37% loss of support), while it takes 30 years to reach the same score for a tie belonging to Tier F+A (3% loss of support).

Here too, these results show a faster rate of degradation and a shorter tie life for the ties with high loss of adjacent support as compared to those with lower loss of adjacent support.

To better understand the different degradations rates, and corresponding different average tie lives, the change in tie score of Tier B+C, Tier A+B+C, and Tier F+A are plotted in Figure 52 below. Regression lines were fitted to the points to model the change of tie condition.

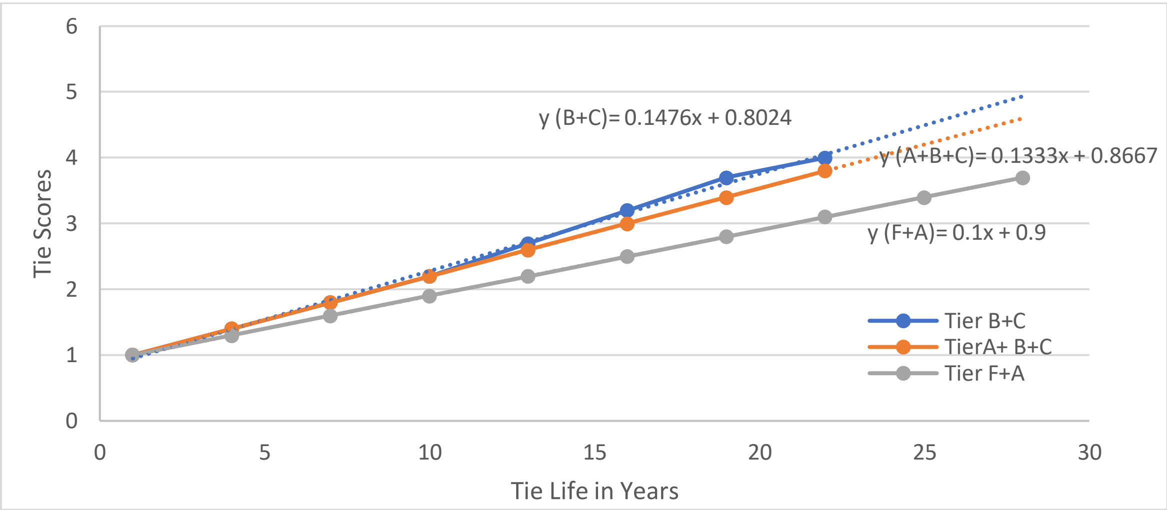


Figure 52: Change in tie score of Tier B+C, Tier A+B+C, and Tier F+A

The resulting regression equations are:

- For F+A: Tie score = 0.1t + 0.9
- For A+B+C: Tie score = 0.1333t + 0.8667
- For B+C: Tie score = 0.1476 t + 0.8024

where t is time in years.

These results again show a faster rate of degradation for the ties with poor adjacent ties (poor support) as compared to all or mainly good ties on either side. Table 34 below summarizes the tie degradation rates for each group.

Table 34: Loss of support and degradation rates for different tiers

Tier	Loss of Support	Tie Degradation Rate
F+A	3%	0.1
A+B+C	19%	0.1333
B+C	37%	0.1476

The tie degradation rate is the slope of the regression line. It should be noted that the tie degradation rate increases as the loss of support due to adjacent tie condition increases from F+A to B+C. Figure 53 shows this change of degradation rate with the increase of loss of support.

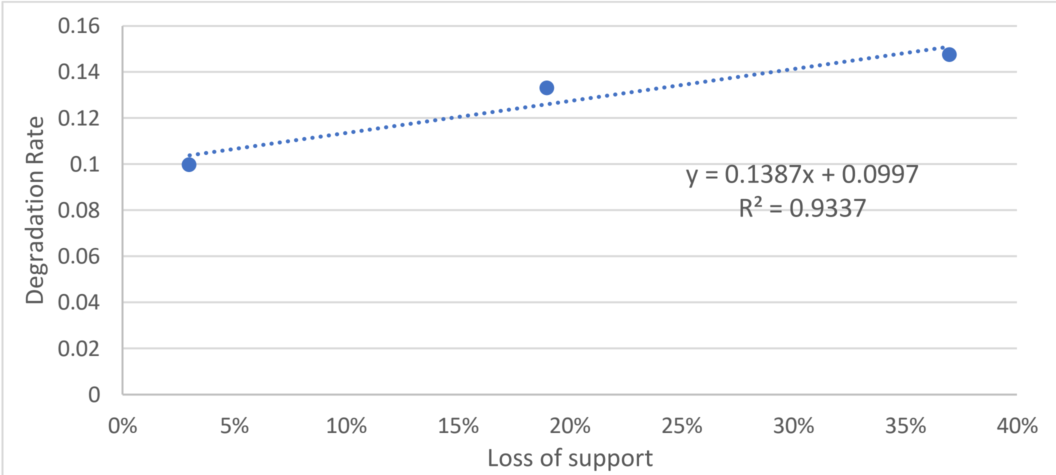


Figure 53: Loss of support Vs Tie degradation rate

From Figure 53, the tie degradation rate increases as the loss of support increases following the equation:

$$\text{Degradation Rate} = 0.1387\ LS + 0.0997$$

where LS is the percent loss of support.

The trend is well defined. As the loss of support increases, corresponding to increasing number of poor ties adjacent to the center tie, the tie condition degradation rate likewise increases. Thus, the poorer the adjacent tie support condition, the faster the rate of tie degradation, and the shorter the average tie life.

8.3. Conclusion

The objective of this section was to provide a way to predict and model tie life based on support condition as defined by the condition of adjacent cross-ties. This analysis made us of a piecewise reconstruction of the tie life as a function of varying support condition as well as the calculation of the rate of tie condition degradation as a function of the support condition, as defined by the adjacent cross-ties. The result was that ties with the greatest loss of support showed shorter predicted average lives as compared to ties where the loss of support was not as significant. Tie condition degradation rates were generated as a function of adjacent tie support condition and associated loss of support due to poor adjacent ties. The resulting degradation rate increased as percentage loss of support increased.

Using a piecewise reconstruction of tie life as a function of varying support condition it was possible to calculate a relationship between percent loss of support and average tie life. In addition, an equation for tie life reduction as a function of adjacent tie condition was also generated.

9. Crosstie Life Piecewise Reconstruction using Dijkstra's Algorithm

After using regression functions to reconstruct an average tie life, an alternate approach was used to reconstruct average tie life and to develop the relationship between tie life and adjacent tie condition in this section. Specifically, Dijkstra's Algorithm, an algorithm for finding the shortest paths between nodes, was be used.

Dijkstra's Algorithm provided a piecewise reconstruction of tie life as a function of varying support condition and a way to calculate a relationship between percent loss of support and average tie life.

9. 1. Dijkstra's Algorithm and Adjacency Matrices

9.1.1. Dijkstra's Algorithm

In 1959, Edsger Dijkstra, a Dutch mathematician, came up with an algorithm to obtain the path of minimum total length between any two vertices belonging to a weighted graph. The graph can either be directed or undirected, and the weights must all be positive.

Dijkstra's algorithm is recursive. It uses the concept that the minimal paths from the initial vertex to the other vertices are built in order of increasing length until the final vertex is reached [20]. In other words, the algorithm is based on finding the length of the shortest path from the starting vertex to all the other connected vertices (one by one) progressively [21].

Dijkstra's algorithm is based on a series of iterations that allows for the identification of the shortest path between nodes; i.e. the minimum length [20, 21, 22].

9.1.2. Adjacency Matrices

An adjacency Matrix can be defined as a matrix that "allows representing a graph with a V × V matrix M = [f (i, j)] where each element f (i, j) contains the attributes of the edge (i, j). If the edges do not have an attribute, the graph can be represented by a Boolean matrix to save memory space" [23].

To be able to apply graph theory to the tie problem being addressed in this report, the first step is to transform the tables in Appendix A, which show the tie condition scores change, into adjacency matrices that would represent distances in a weighted graph. The nodes or vertices of the graph will be represented by different tie scores: 1, 1.1, 1.2…3.9, and 4. The objective is to use algorithms to minimize the path from Vertex 1 ("new" tie) to Vertex 4 ("failed" tie), corresponding to the change in tie condition over time. Thus, Vertex 1 corresponds to the condition 1.0, Vertex 1.1 to condition 1.1, Vertex 3 to condition 3., and Vertex 4 to condition 4.

To accomplish this, first, the percentage of ties changing conditions (from their initial condition score) within the 3 years (2016- 2019) is calculated as illustrated in Equation 2.

As an example, the percentages calculated using Equation 2 for tie scores 1.0 to 2.0 in group F are presented in Table 35.

Table 35: Example of percentage of tie score changes

	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
1	15.94%	17.91%	11.70%	7.71%	13.51%	6.45%	3.14%	3.50%	3.57%	4.69%	2.15%
1.1		18.54%	12.37%	8.36%	17.41%	8.70%	4.05%	4.54%	5.16%	6.71%	2.19%
1.2			12.87%	9.11%	18.42%	10.24%	5.00%	6.10%	6.72%	10.26%	3.20%
1.3				9.29%	18.35%	10.19%	6.03%	6.89%	8.92%	13.72%	3.90%
1.4					16.12%	10.25%	6.39%	8.29%	9.33%	16.66%	5.91%
1.5						11.19%	6.57%	8.53%	10.91%	17.68%	5.36%
1.6							5.33%	7.22%	9.03%	17.30%	4.09%
1.7								5.57%	8.09%	16.82%	4.78%
1.8									7.16%	16.45%	4.41%
1.9										16.29%	6.32%
2.0											12.45%

Based on the percentage calculated, the weights can then be defined as follows in Equation 11:

$$Weight = \frac{1}{Percentage} \qquad \textit{Equation 11}$$

The adjacent matrix weights represent the distance between nodes on the graph. The minimized sum of the weights represents the minimum path. This is identical to the maximum path considering the percentages.

The tie score changes or jumps represent the deterioration behavior that the majority (high percentage) of ties undergo. In other words, the resulting path from the Dijkstra’s algorithm application would represent the average tie life that the majority of cross-ties in a group or tier have. This method allows for modelling and predicting the average tie life.

The higher the percentage of ties having the same initial score and having the same final one, the higher the likelihood that a random tie having that initial score will have the final score in 3 years. Except in extreme conditions (such as a breaking), tie condition degradation happens gradually. Ties showing a high score change in three years represent exceptions (for instance a change from 1 to a score of 3.5 within 3 years). In order to avoid unrealistic tie changes, the adjacency matrix limits possible tie score changes to a maximum range of 0.6 (for instance from 1 to 1.6, or from 1.3 to 1.9). Unchanged tie scores (from 1.0 to 1.0, or 1.1 to1.1…) were represented by a distance that equals 0.

Using Equation 11 on the percentages presented in Table 35 gives the adjacency matrix as shown in Table 36.

Table 36: Example of an adjacency matrix

	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0

1.0	0.00	5.58	8.55	12.97	7.40	15.51	31.88				
1.1		0.00	8.08	11.95	5.74	11.49	24.70	22.01			
1.2			0.00	10.98	5.43	9.77	20.00	16.38	14.88		
1.3				0.00	5.45	9.81	16.60	14.52	11.21	7.29	
1.4					0.00	9.75	15.66	12.06	10.71	6.00	16.92
1.5						0.00	15.21	11.72	9.16	5.66	18.65
1.6							0.00	13.85	11.07	5.78	24.43
1.7								0.00	12.36	5.95	20.93
1.8									0.00	6.08	22.67
1.9										0.00	15.81
2.0											0.00

9.2. Application of Dijkstra's Algorithm

9.2.1 Comparing Each Tier Independently

After generating the adjacency matrices, representative weighted graphs are generated for each group or tier such that the vertices are represented by decimal tie scores from 1, 1.1, 1.2, … to 4. The edges are then weighted, and these represent the "distances" between the tie condition scores. The adjacency matrices for each group and tier were then converted into a graph[9][24, 25], as shown in Figure 54:

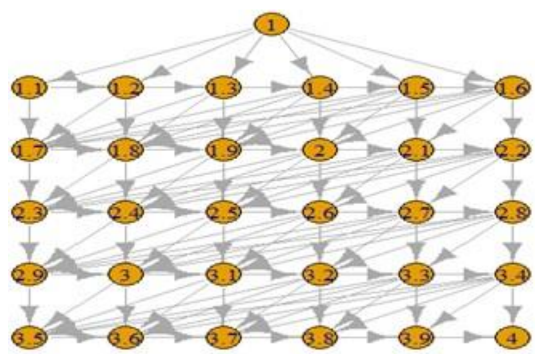


Figure 54: Graph representation of the tie score changes

The Dijkstra algorithm was then applied to each representative graph in order to find the shortest path from Vertex = 1 ("new" tie) to Vertex = 4 ("failed" tie). The graphs for Groups F, A, B and C are shown in Figures 55a through 55d. The orange path represents the shortest path from Vertex 1 to Vertex 4 using Dijkstra's algorithm:

[9] This was done using the R open source environment. Graphs were generated using the igraph and DiagrammeR packages.

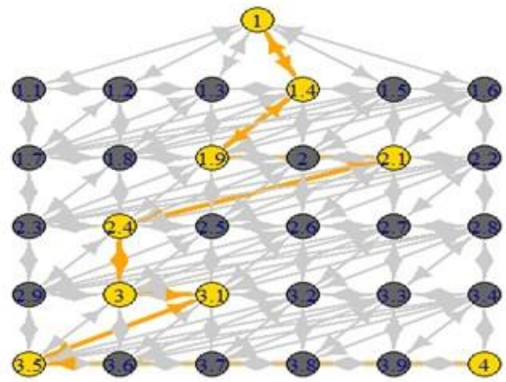


Figure 55a: Shortest path for group F

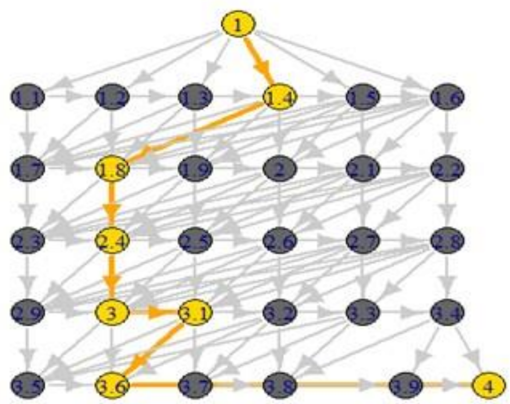


Figure 55b: Shortest path for group A

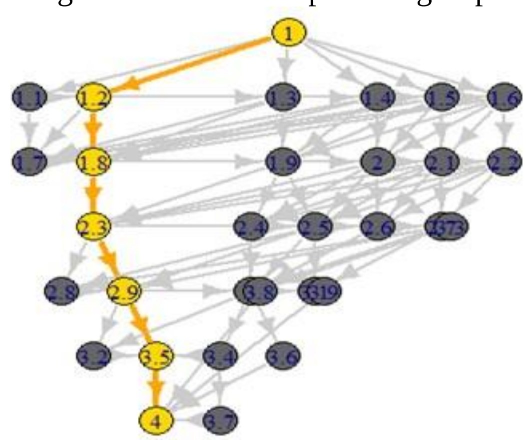


Figure 55c: Shortest path for group B

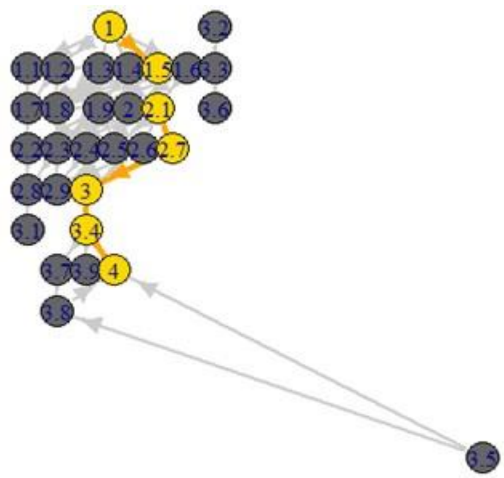


Figure 55 d: Shortest path for group C

The weights used in the graph represents the inverse of percentages of tie score changes. Thus, the shortest path represents the sequence of tie scores (vertices) that the majority of ties in a group will go through before tie failure (i.e. score = 4.0). These sequences of tie scores can be used to forecast the tie life. Because the dataset represents inspections from 2016 and 2019, every step in the short path will be represented by a period of 3 years.

Assuming a tie score of 1 will occur in year 1, the tie condition changes are plotted in Figure 56 for each group. Note, each data point in the figure corresponds to a step in the Dijkstra's algorithm graph representation and is equivalent to an interval of 3 years.

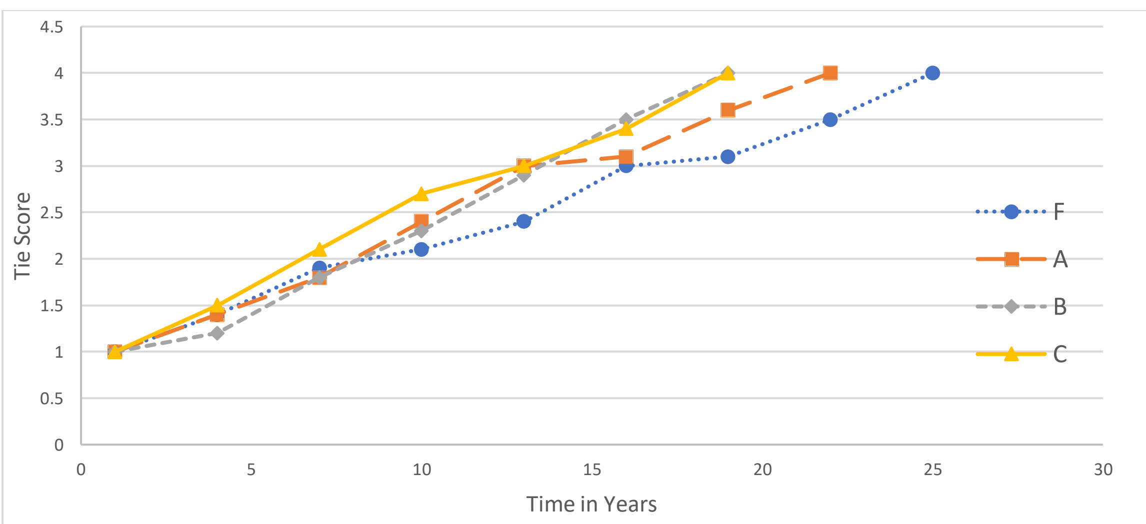


Figure 56a: Time in years vs Tie Score for group A, B, C, and F

Figure 56a shows the forecast tie score changes from 1 to 4 for each group of ties. Note, the tie has failed when it reached condition 4, thus the end of its life. Further note that the tie life is longest for ties with good support, and shortest for ties with poor support.

A regression fit was performed on the data in Figure 56a to obtain the degradation rates for each group. By setting the intercept to 1 (new tie) in Figure 56b for all the groups the regression generates the following equations:

- For group F: tie score = 0.116t + 1
- For group A: tie score = 0.1366t + 1
- For group B: tie score = 0.1482t + 1
- For group C: tie score = 0.1556t + 1

where t is time in years. Note, the tie degradation rate is the slope of the regression line. It should be further noted that the tie degradation rate increases as the loss of support due to adjacent tie condition increases from F to C.

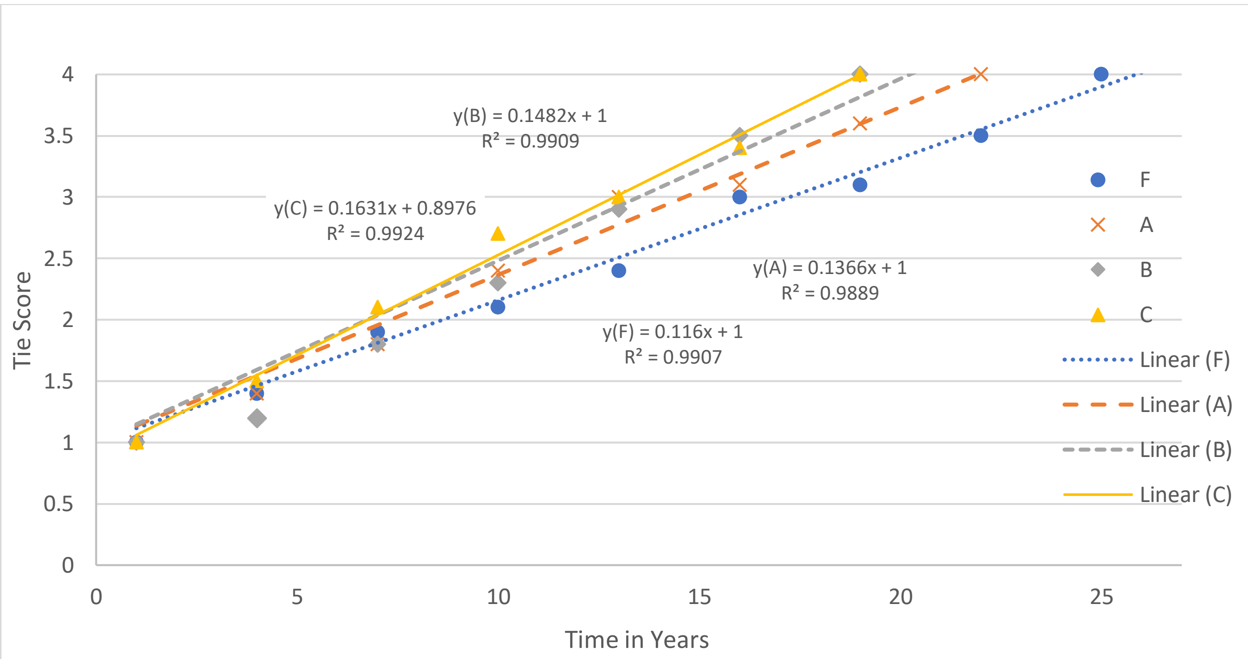


Figure 56b: Tie score change over time- Linear modeling

9.2.2 Bundling Groups

Because the dataset is unbalanced, i.e. having significantly different number of ties in each category (see Table 13), the smaller number of ties in groups B and C, groups were bundled together. Three different approaches were used.

In the first approach, three tiers were created corresponding to groups F, A, and B+C. These are shown in Figures 55a, 55b and 57a respectively.

In the second, two tiers were created corresponding to groups F+A, and B+C. These are shown in Figures 57b, and 57a respectively.

In the third, two tiers were created corresponding to groups F and A+B+C. These are shown in Figures 55a and 57c respectively.

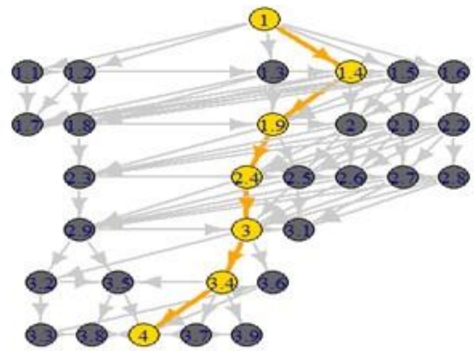


Figure 57a: Shortest path for Tier B + C

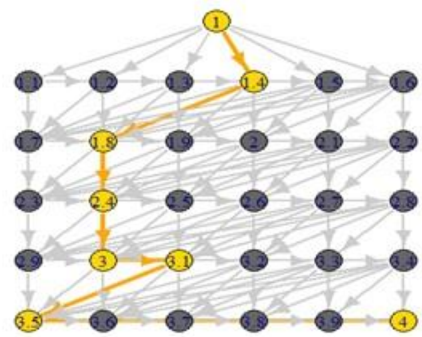


Figure 57b: Shortest path for Tier F+A

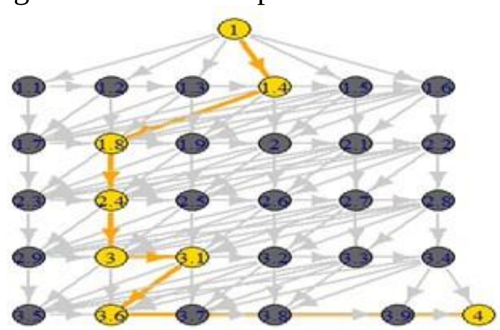


Figure 57c: Shortest path for Tier A+B +C

Bundling: F vs A vs B+C

Because of the low number of ties in group B and C, and because the same average tie life (19 years) resulted from the Dijkstra's algorithm, these two groups were bundled together and compared to Groups A and F.

Figure 58a shows how the degradation of the tie condition over time behaves for these three groups F, A, and B+C. Again, note that each data point in the figure corresponds to a step in the Dijkstra's algorithm graph representation and is equivalent to an interval of 3 years.

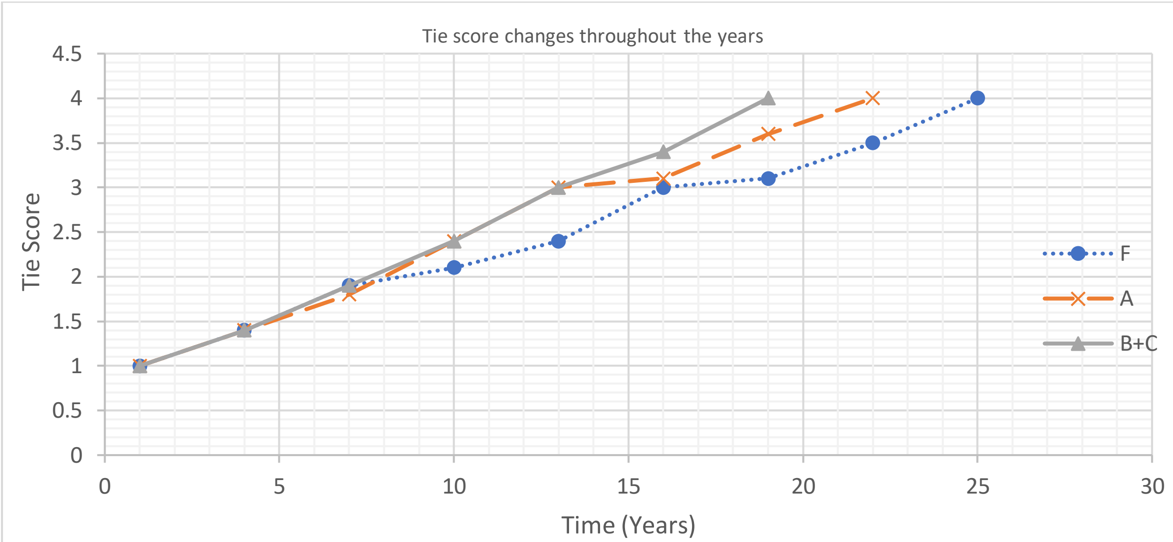


Figure 58a: Time in years vs tie score for each group

A regression line was fit to these data points to model the change of tie score (tie condition) throughout the life of the tie for all three cases as shown in Figure 58b.

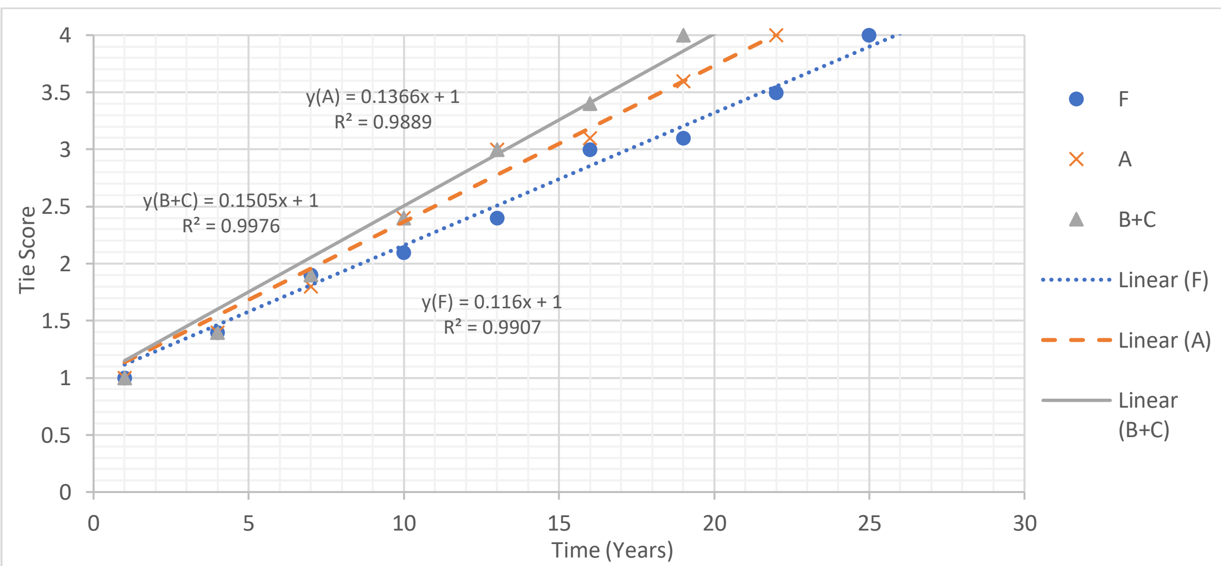


Figure 58b: Tie score change over time - linear modelling (Tier B+C)

Once again, regression lines were generated, setting the intercept to 1, with the equations resulting as follows:

- For group F: tie score = 0.116t + 1
- For group A: tie score = 0.1366t + 1
- For group B+C: tie score = 0.1505t + 1

where t is time in years. Again, note the increase in slope, corresponding to an increase in the rate of tie condition degradation, as the adjacent tie support condition degrades (i.e. from F to A+B).

Bundling: F vs A+B+C

In a similar manner, the three groups A, B, and C (ties having adjacent tie loss of support) are bundled and compared to group F which represents the best-case scenario (no loss of support).

Figure 59 shows the results of applying Dijkstra's algorithm on the changing tie condition scores for these two cases, good adjacent tie condition and deteriorated adjacent tie condition (A+B+C).

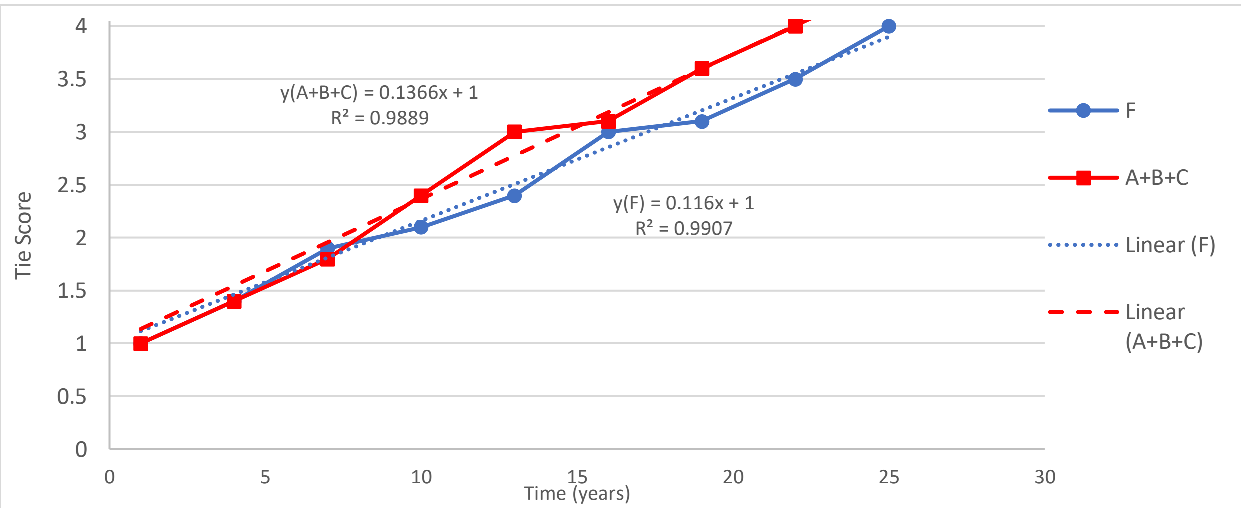


Figure 59: Time in years vs tie score for Group F and Tier A+B+C

It can be seen that group F and group A+B+C behave similarly until condition score 2 is reached. Then, Tier A+B+C shows a faster rate of degradation (and shorter tie life) than Group F, with an average tie life for Tier A+B+C of 22 years, as compared to 25 years for group F.

Again, a regression line was fitted to the points to model the change of tie condition.
The resulting regression equations are:

- For group F: Tie score = 0.116t + 1
- For A+B+C, Tie score = 0.1366t + 1

where t is time in years.

These results likewise show a faster rate of degradation for the ties with poor adjacent ties (poor support) as compared to all good ties on either side Thus, the rate of degradation (slope) of 0.116 for good adjacent tie condition (Group F) is smaller than the rate of degradation (slope) of 0.1366 for poorer adjacent tie condition (Tier A+B+C).

Bundling: F+A vs B+C

The final bundling approach consisted of two groups, where F and A are bundled and compared to Tier B+ C.

Figure 60 shows the degradation of the tie condition over time for these two groups F+A, and B+C. Again, note that each data point in the figure corresponds to a step in the Dijkstra's algorithm graph representation and is equivalent to an interval of 3 years.

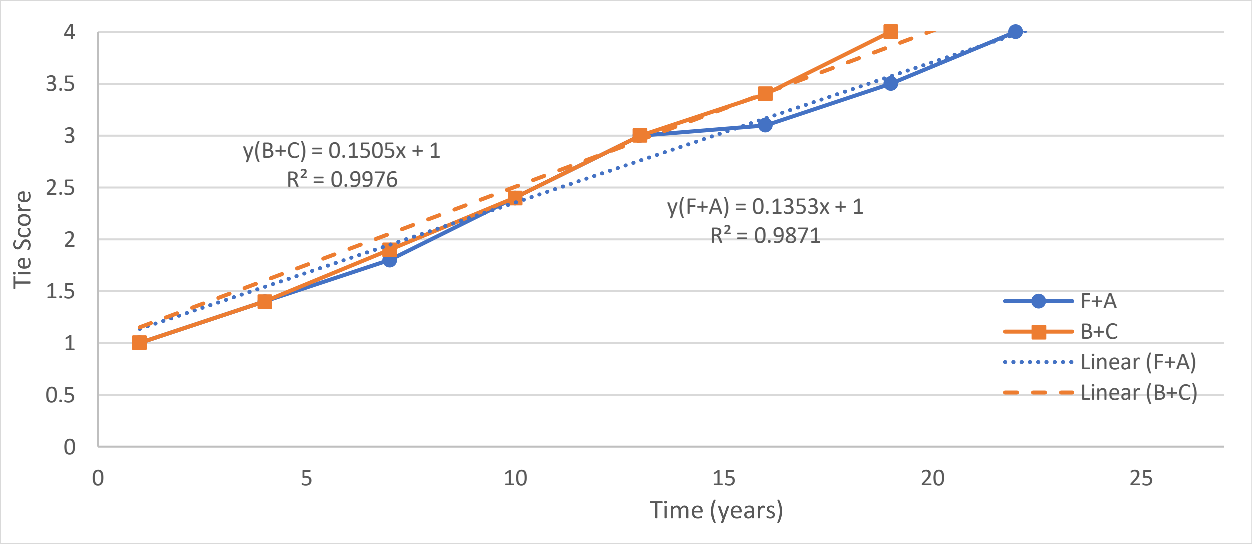


Figure 60: Time in years vs tie score for Tier F+A and Tier B+C

In this case, both tiers F+A and B+C behave similarly until a score 3 is reached. Then, Tier B+C starts having a higher rate of degradation than the better supported Tier F+A with an average tie life for Tier B+C of 19 years, as compared to 22 years for group F.

Once again a regression line was fitted to the points to model the change of tie condition score (tie condition) with the resulting equations:

Tier F+A : Tie score = 0.1353t + 1
Tier B+C : Tie score = 0.1505t + 1

where t is time in years.

Here too, these results show a faster rate of degradation for the ties with poor adjacent ties (poor support) as compared to better tie support. The rate of degradation (slope) of 0.1353 for good adjacent tie condition (Group F+A) is smaller than the rate of degradation (slope) of 0.1505 for poorer adjacent tie condition (Tier B+C).

9.3. Comparisons and Analysis

9. 3. 1. Degradation Rates Comparison

Using the slope of the calculated regression equations, presented previously, it is possible to evaluate the corresponding degradation rates of the different data sets associated with varying degrees of support condition, as defined by the condition of adjacent (support) ties. These degradation rates, corresponding to the slope of the tie condition-time curves presented above, are summarized in Table 37 as a function of support condition, by group. Note, the percent loss of support shown in Table 37, is calculated from the BOEF equation for the corresponding adjacent tie support conditions associated with each group[10].

Table 37: Degradation Rates

Group	Percent loss of support	Degradation Rate
F	0	0.116
F+A	3	0.1353
A	16.67	0.1366
A+B+C	19	0.1366
B	33	0.1482
B+C	37	0.1505
C	46	0.1556

Using these values, tie condition degradation rate can be plotted against the percent loss of support as presented in Figure 61.

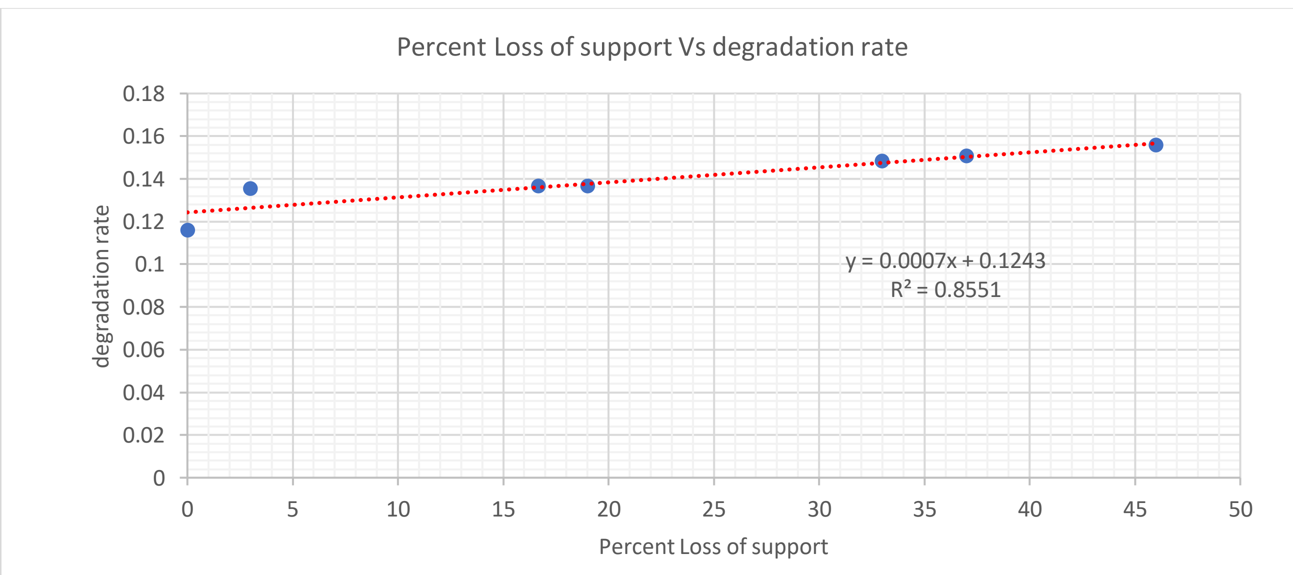


Figure 61: Percent loss vs degradation rate

[10] For combined groups, the loss of support is calculated using a weighted average of the individual group loss of support values.

The trend is clear and well defined. As the loss of support increases, corresponding to increasing number of poor ties adjacent to the center tie, the tie condition degradation rate likewise increases. Thus, the poorer the adjacent tie support condition, the faster the rate of tie degradation, and the shorter the average tie life. This is consistent with the results presented in Reference 9.

9. 3. 2. Comparison with Previous Study[11]

As noted, the rate of tie condition degradation is directly related to the support condition associated with the adjacent tie condition. This was presented in Reference 9 using a simplified analysis approach, hereinafter referred to as Method A, and in this section using a more sophisticated and accurate approach, based on the Dijkstra method, hereinafter referred to as Method D. In addition, the results presented here show a calculated tie life, as a function of support condition. Using this calculated tie life, it is possible to determine the reduction in tie life as a function of loss of tie support condition. This will be referred to here as the life reduction factor.

The life reduction factor is a measure of how much the tie life is reduced as a function of percent loss of support. As such it is the ratio of average life for each tie support condition group (e.g. A, B, C) as compared to the fully supported case (F, all good adjacent ties) where the loss of support is 0 %. This is summarized in Table 38.

The life reduction factor for any Group I is defined in Equation 12.

$$Life\ reduction\ factor\ (Group\ I) = \frac{Average\ LifeGroup\ I}{Average\ life\ of\ (F)} \qquad \text{Equation 12}$$

Table 38: Life reduction factor using Method D and Method A for groups A,B,C, and F

Tiers	Loss of support	Tie life using Dijkstra	Dijkstra based (Method D) Life reduction factor	Previous study (Method A) life reduction factor
F	0%	25	1.00	1.00
A	17%	22	0.88	0.79
B	33%	19	0.76	0.74
C	46%	19	0.76	0.68

These results are shown graphically in Figure 62. As can be seen in Table 38 and Figure 62, the life reduction factor based on Dijkstra's method (Method D) is less severe than the life reduction factor calculated using the previous simplified analysis approach (Method A). This indicates that the effect of adjacent tie condition and associated loss of support is somewhat smaller than the simpler Method A analysis suggests. In fact, this may be more realistic, since the Method A formula shows a dramatic reduction in tie life associated with modest loss of tie support (21%), as compared to a more realistic reduction associated with the Method D approach of only 12%.

[11] The previous study referred to here is the one presented in Reference 9 at the 2020 annual AREMA (American Railway Engineering and Maintenance of way Association) conference.

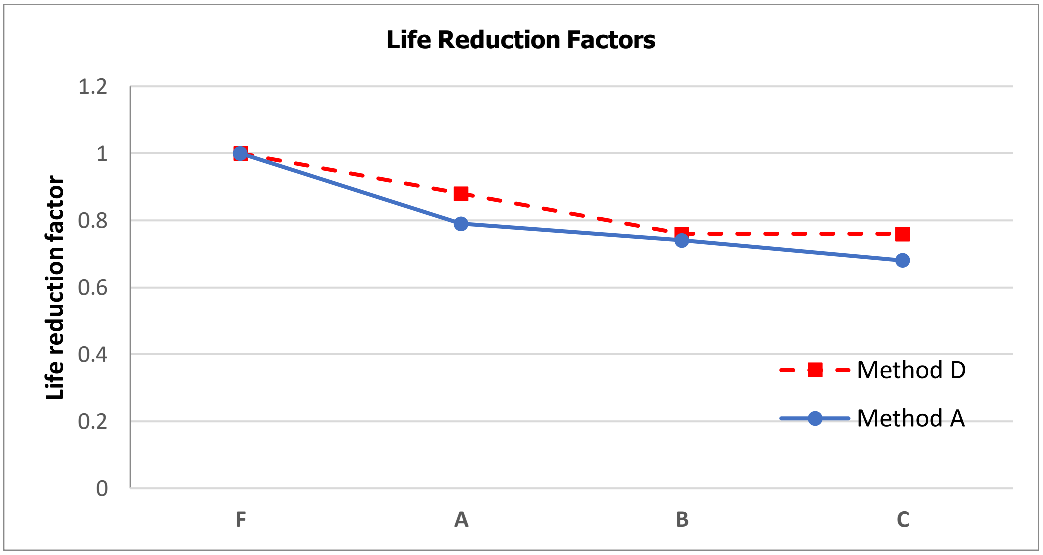


Figure 62: Life reduction factors using Method D and Method A

Expanding the comparison to include the intermediate support conditions represented by the "bundling" presented previously results in the tie lives and associated tie life reduction factors presented in Table 39 and Figure 63.

Table 39: Life reduction factor using Method D and Method A for all groups and tiers

	Weighted average Loss of support (%)	**Tie life using Method D**	**Tie life using Method A**	**Method D life reduction factor**	**Method A life reduction factor**
F	0	25	25	1.00	1.00
F+A	3%	22	23.87	0.88	0.95
A	16.70%	22	20.31	0.88	0.81
A+B+C	19%	22	19.85	0.88	0.79
B	33%	19	17.84	0.76	0.71
B+C	37%	19	17.53	0.76	0.70

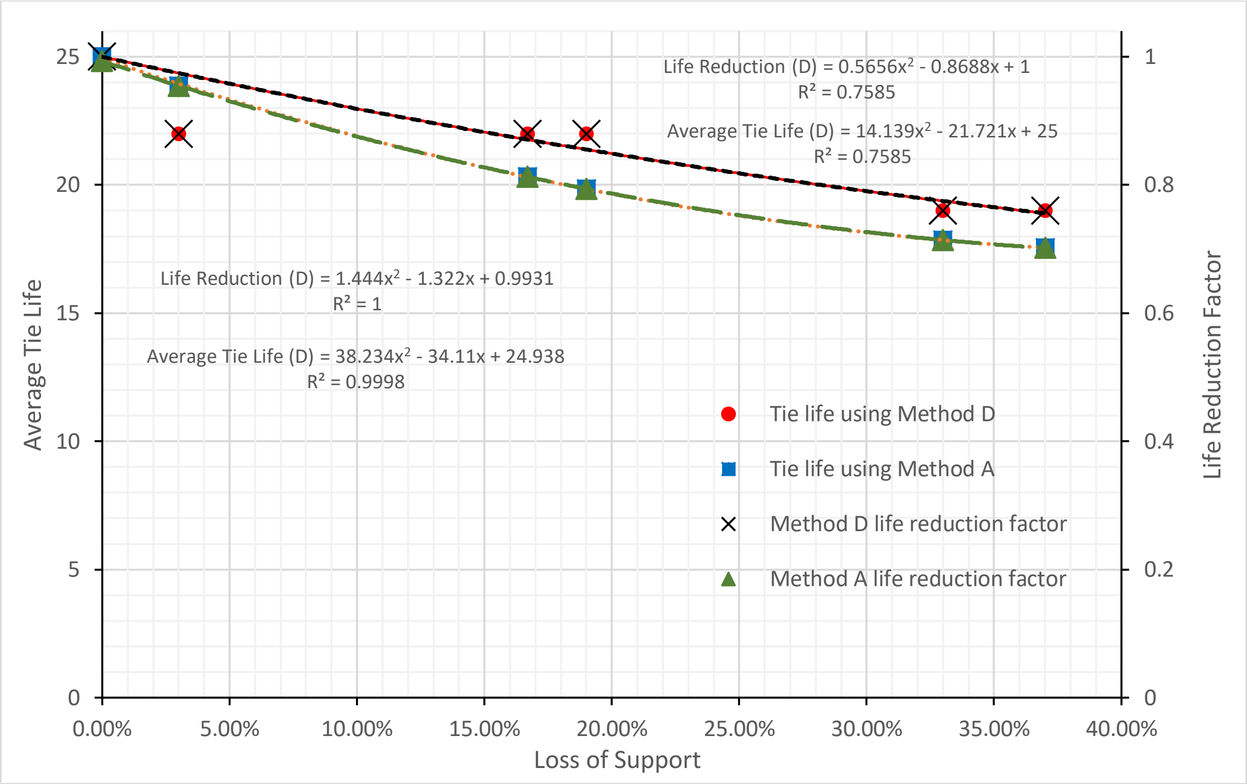


Figure 63: Loss of support vs average tie life and life reduction factor using Method D and Method A

From Figure 63, it is possible to calculate a relationship between percent loss of support and average tie life, using a base case life of 25 years for a loss of support of 0%. The resulting equations, based on a quadratic regression are as follows:

Method D average tie life formula	Average tie life = 14.139 LS^2 -21.721LS+25
Method A average tie life formula	Average tie life =38.134 LS^2-34.11LS +24.938

where LS is the % loss of support associated with the adjacent tie condition.

Furthermore, from Figure 63, a similar relationship can be obtained for the tie life reduction factor as a function of percent loss of support. These are as follows:

Method D life reduction formula	Life Reduction = 0.5656 LS^2 - 0.8688 LS + 1
Method A life reduction formula	Life Reduction = 1.444 LS^2 - 1.322 LS + 0.9931

where LS is the % loss of support associated with the adjacent tie condition.

9.4. Conclusion

The objective of this activity was to provide a way to predict and model tie life based on support condition as defined by the condition of adjacent cross-ties. This included a piecewise reconstruction of tie life as a function of varying support condition using Dijkstra's theorem as well as the calculation of the rate of tie condition degradation as a function of the support condition defined by the adjacent cross-ties. The study represented an extension of an earlier study where a simplified regression analysis approach was used to calculate tie life reduction. In addition to using a more accurate and effective modeling approach, this study also was able to calculate the actual average tie life as a function of support condition.

These tie score changes were then used to develop an adjacency matrix and associated weighted graphs, where the vertices corresponded to different tie scores. Dijkstra's algorithm was then applied to find the shortest path from Vertex 1 (best condition) to Vertex 4 (worst/failed condition) for the different groups and tiers (combined data sets) with differing loss of support associated with the condition of the adjacent ties. The result was that ties with the greatest loss of support showed shorter predicted average lives as compared to ties where the loss of support was not as significant.

Using the results of the Dijkstra analysis, tie condition degradation rates were generated as a function of adjacent tie support condition and associated loss of support due to poor adjacent ties. The resulting degradation rate increased as percentage loss of support increased.

Using the piecewise reconstruction of tie life as a function of varying support condition based on Dijkstra's theorem, it was possible to calculate a relationship between percent loss of support and average tie life. In addition, using these generated tie lives as a function of loss of support condition, an equation for tie life reduction was also generated.

This life reduction factor based on the Dijkstra's (Method D) method is less aggressive than the life reduction factor calculated using the previous study (Method A). This indicates, that the effect of adjacent tie condition and associated loss of support is somewhat smaller than the simpler Method A analysis, and in fact may be more realistic. The formula from Method A shows a dramatic reduction in tie life associated with modest loss of tie support (21%), as compared to a more realistic reduction associated with the revised (Method D) approach of only 12%.

However, despite their different degradation rates, both the Method A formula and the Method D formula developed here-in confirm the fact that loss of adjacent tie support contributes to the premature degradation and failure of a tie.

10. Markov Chain Application for Tie Failure Prediction

As a second alternative analysis approach, Markov chains were used to determine tie life and the effect of loss of adjacent tie support. Markov chains also allow for the prediction of the change of probability of reaching a particular tie condition over time.

Markov chains represent a process in which the outcome of a particular event is influenced by that of a previous event [26]. In fact, they are a sequence of random variables represented by discrete states such that the state at time 't' depends on the state at time 't-1' [27]. In this study, each tie score will be represented by a Markov state, and the transitions (from one state to the other) represent the probabilities.

Let P_{ij} be the probability of a Markov chain process to transition from state j to i [28].

Knowing that there are N discrete states, the Markov chain one step transition matrix can be defined as follow [28]:

$$P = \begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \cdots & \mathrm{P}_{NN} \end{bmatrix}$$

Furthermore, the sum of states transitions probability is equal to 1 as follow:

$\sum_{i=0}^{N} Pij = 1 \quad , \; for\; j = \; 0, 1, \ldots, N$ [24]

$P_{ij}^{(n)}$ is the n-step transition probability of a Markov chain process. In other words, $P_{ij}^{(n)}$ represents the probability that after n steps, an event in state j would be in i [28].

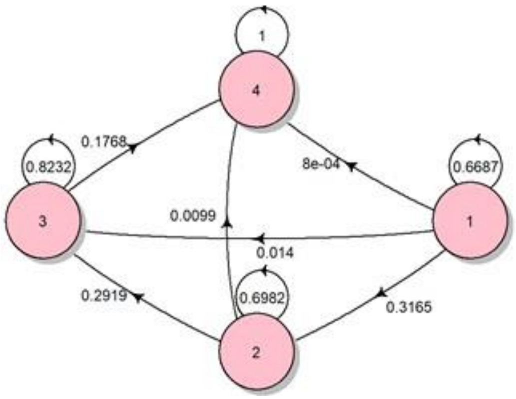
Markov Transition Matrices were generated for each group using the transition probabilities. It is to be noted, that in this section, the assigned tie scores were grouped into integers (1, 2, 3, and 4)) and every tie score was represented by a discrete Markov state. To build the transition matrices, probabilities were represented by the percentage of ties changing condition (from their initial discrete condition score) within the 3 year (2016- 2019) period. The percentage of ties changing conditions within the 3 years period was calculated using Equation 2.

The transition matrices representing tie condition states for groups F, A, B, and C can be seen in Tables 40A, 40B, 40C, and 40D respectively.

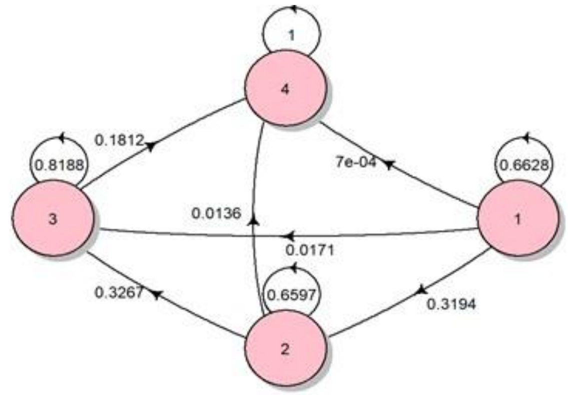
Table 40: Transition Matrices

A: Markov Transition Matrix for F						B: Markov Transition Matrix for A				
	1	2	3	4			1	2	3	4
	0.6687	0.3165	0.0140	0.0008		1	0.6628	0.3194	0.0171	0.0007
2	0.0000	0.6982	0.2919	0.0099		2	0.0000	0.6597	0.3267	0.0136
3	0.0000	0.0000	0.8232	0.1768		3	0.0000	0.0000	0.8188	0.1812
4	0.0000	0.0000	0.0000	1.0000		4	0.0000	0.0000	0.0000	1.0000
C: Markov Transition Matrix for B						D: Markov Transition Matrix for C				
	1	2	3	4			1	2	3	4
	0.5991	0.3645	0.0330	0.0034		1	0.5901	0.3778	0.0296	0.0025
2	0.0000	0.6777	0.3071	0.0152		2	0.0000	0.6870	0.3043	0.0087
3	0.0000	0.0000	0.8359	0.1641		3	0.0000	0.0000	0.7963	0.2037
4	0.0000	0.0000	0.0000	1.0000		4	0.0000	0.0000	0.0000	1.0000

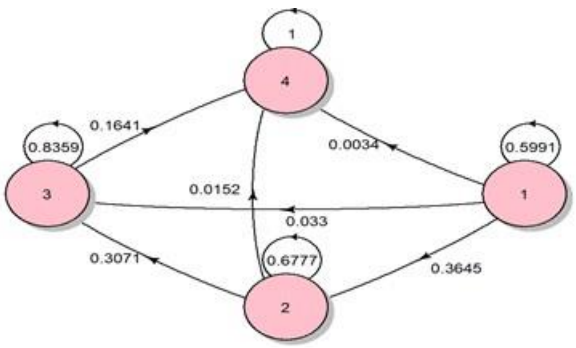
The equivalent Markov Diagrams for groups F, A, B, and C were generated using R software [28] and are illustrated in Figures 64



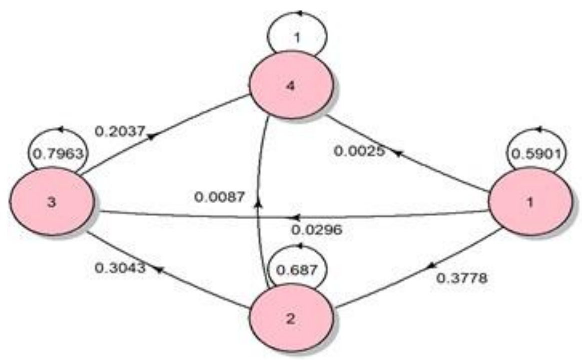
A: Markov Graph -Group F



B: Markov Graph -Group A



C: Markov Graph -Group B



D: Markov Graph -Group C

Figure 64: Markov Graphs

10.1. Markov Chain Results

Using the matrices generated in the previous section, a step chain probability prediction was performed in R software [28] in order to determine the change of failure probability as a function of chain steps. It is to be noted that failure is defined as a tie reaching a score of either 3 or 4. The initial state of each Markov chain was set as the initial tie condition score for each group as 1 (best condition).

Markov models were generated for each group independently over 25 chain steps (iterations) to determine the probability of reaching a specific tie condition score change over the chain iterations. Chain steps represent time.

The change of probability to reach score 1 or 2 (probability to reach 1 + probability to reach 2) is represented by the blue plot in Figure 65. The change of probability to reach score 3 or 4 representing tie failure (probability to reach 3 + probability to reach 4) is represented by the red dashed plot in Figure 65. Note that the solid lines represents good tie condition scores (1 or 2), and the dashed lines represent bad tie condition scores (3 or 4).

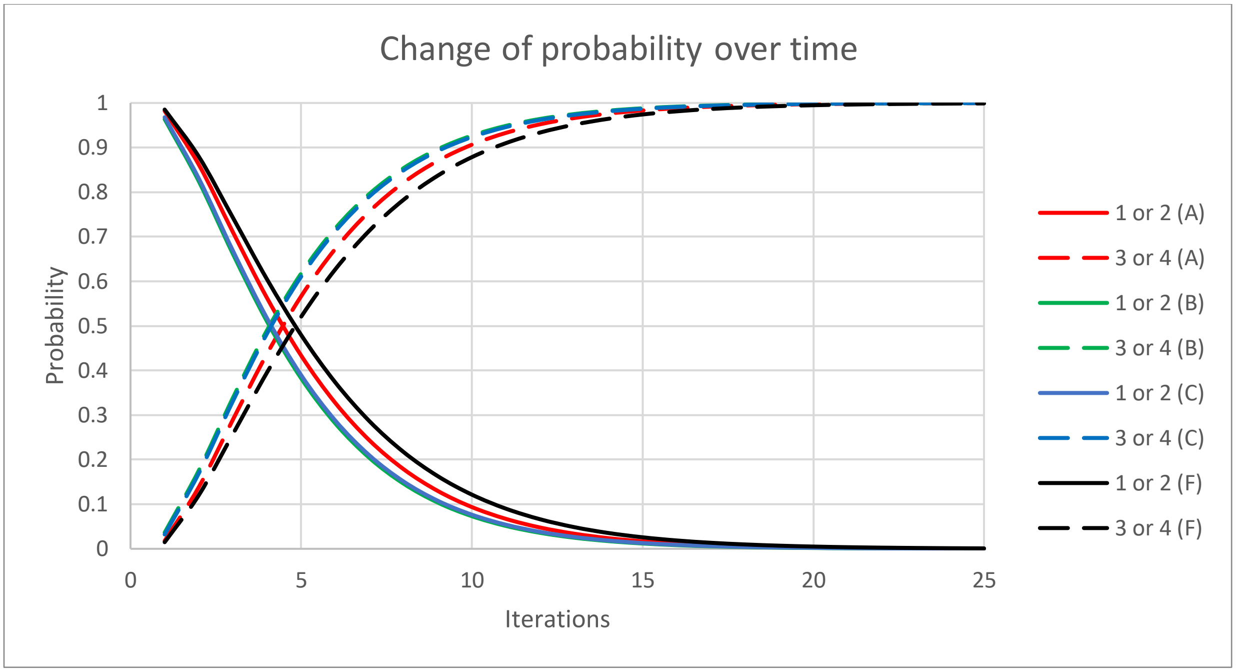


Figure 65: Change of tie score probability over time (chain iterations) for groups A, B, C, and F

From Figure 65, it can be seen that the change in failure probability differs from one group to the other. For a similar number of iterations (i.e. time), failure is more likely to occur for ties belonging to groups with higher loss of support. In other words, the likelihood of failure grows as ties lose adjacent support.

Setting the failure probability threshold to 75% and to 90 % respectively, the chain steps necessary to reach the thresholds were counted for each group and are summarized in Table 41. A life reduction factor was then determined using Equation 12.

Table 41: Markov Chains Life Reduction Factor

Group	**Percent loss of support**	**Probability of 75%**		**Probability of 90%**	
		Chain steps	Life reduction factor	Chain steps	Life reduction factor
F	0%	7.5	1	10.5	1
A	16.67%	7	0.93	10	0.95
B	33%	6.5	0.87	9	0.86
C	46%	6.5	0.87	9	0.86

It can be seen that the life reduction factor decreases as the percent loss of support increases. The life reduction factor is plotted against the loss of support, and can be seen in Figure 66 below:

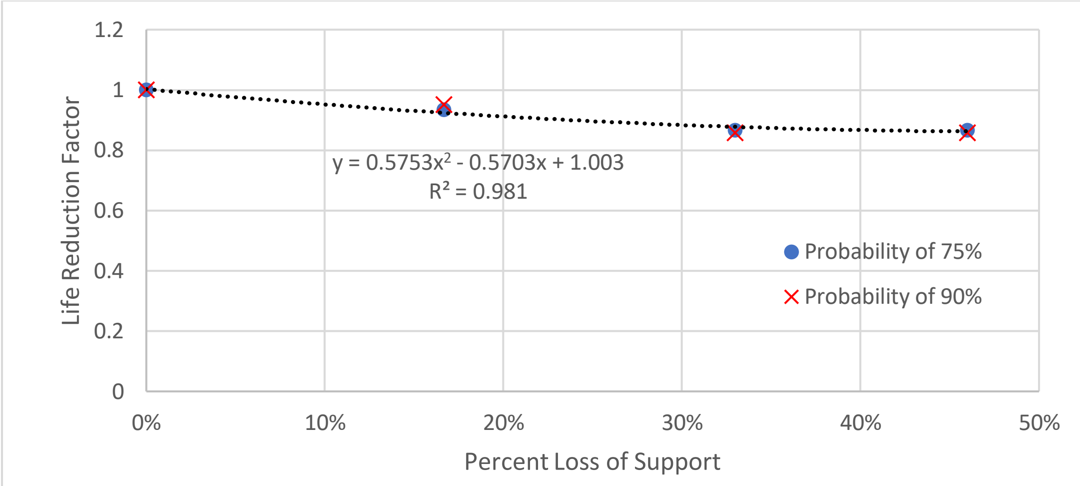


Figure 65: Life reduction factor as a function of loss of support

From Figure 66 above, the life reduction factor can be seen to be behaving similarly for both probability thresholds. The trend is well defined, the higher the loss of support, the lower the life reduction factor, and thus the lower the tie life. The life reduction factor change as a function of percent loss of support can be defined by the following the equation:

$$\text{Life Reduction Factor} = 0.5753\ LS^2 - 0.5703\ LS + 1.003$$

10.2. Summary of Analysis Results

As noted, the rate of tie condition degradation is directly related to the support condition associated with the adjacent tie condition. This was first presented in Reference 9 using a simplified analysis approach[12], hereinafter referred to as Method A. In this paper, this relationship was developed using two more sophisticated and accurate approaches, one based on the Dijkstra method, hereinafter referred to as Method D, and the other on Markov Chains, hereinafter referred to as Method M. In addition, the results presented here show the calculated tie life, as a function of adjacent tie support condition. Using this calculated tie life, it is possible to determine the reduction in tie life as a function of loss of adjacent tie support condition. This will be referred to here as the life reduction factor.

The life reduction factor is a measure of how much the tie life is reduced as a function of percent loss of support. As such it is the ratio of average life for each tie support condition group (e.g. A, B, C) as compared to the fully supported case (F, all good adjacent ties) where the loss of support is 0 %. This is summarized in Table 42. The life reduction factor for any group is defined in Equation 12.

[12] The previous study referred to here is the one presented in Reference 9 at the 2020 annual AREMA (American Railway Engineering and Maintenance of way Association) conference.

Table 42: Life reduction factor using Method D and Method A for groups A,B,C, and F

Tiers	Loss of support	Tie life using Dijkstra	Dijkstra based (Method D) Life reduction factor	Markov based (Method D) Life reduction factor	Previous study (Method A) Life reduction factor
F	0%	25	1.00	1.00	1.00
A	17%	22	0.88	0.93	0.79
B	33%	19	0.76	0.87	0.74
C	46%	19	0.76	0.87	0.68

These results are shown graphically in Figure 67. As can be seen in Table 42 and Figure 67, the life reduction based on Dijkstra's algorithm (Method D) is less severe than the life reduction calculated using the previous simplified analysis approach (Method A). Furthermore, the life reduction factor based on Markov Chains (Method M) is less severe than the life reduction factor calculated using both Method A and D. This indicates that the effect of adjacent tie condition and associated loss of support is somewhat smaller than the simpler Method A analysis suggests. In fact, this may be more realistic, since the Method A formula shows a dramatic reduction in tie life associated with modest loss of tie support (21%), as compared to a more realistic reduction associated with the Method D approach of only 12%, and Method M approach of about 7%.

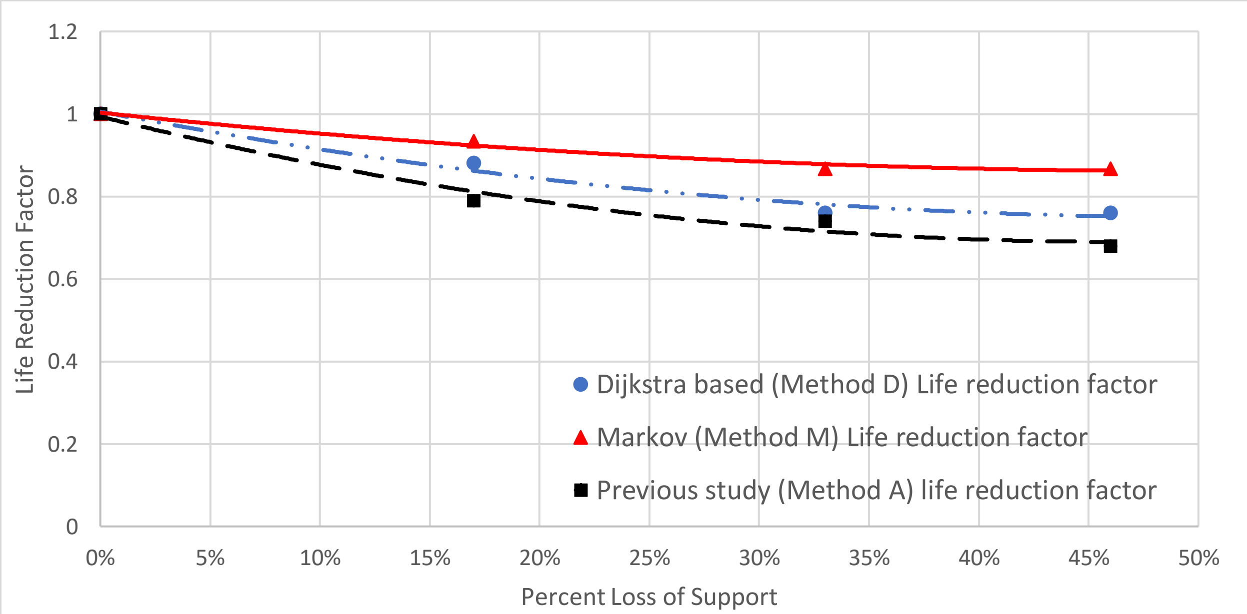


Figure 66 : Life reduction factors using Method D, Method M, and Method A

From Figure 67, it is possible to calculate a relationship between percent loss of support and average tie life, using a base case life of 25 years[13] for a loss of support of 0%. The resulting equations, based on a quadratic regression are as follows:

Method D average tie life formula	Average tie life = 14.139 LS^2 -21.721LS+25
Method M average tie life formula	Average tie life = 13.956 LS^2 -14.075LS+25.076
Method A average tie life formula	Average tie life =38.134 LS^2-34.11LS +24.938

where LS is the % loss of support associated with the adjacent tie condition.

Furthermore, from Figure 67, a similar relationship can be obtained for the tie life reduction factor as a function of percent loss of support as follows:

Method D: Dijkstra life reduction formula	Life Reduction = 1.005 LS^2 – 1.013 LS + 1.005
Method M: Markov Chain Life reduction formula	Life Reduction =0.5753 LS^2 -0.5703 LS + 1.003
Method A (Previous Study) life reduction formula	Life Reduction = 1.444 LS^2 - 1.322 LS + 0.9931

10.3. Conclusion

The objective of this activity was to develop and implement a second methodology to predict tie life based on support condition, as defined by the condition of adjacent cross-ties. In addition, this activity also examined the effect of any loss of support on the tie life and developed a life reduction factor based on loss of adjacent support.

Markov chains were used to model how the probability of premature tie failure changes over time. It was concluded that the probability of tie failure grows faster over time for groups with higher loss of adjacent tie support compared to well supported tie groups.

A life reduction factor was generated for the Markov chain approach together with a life reduction formula as follows:

Life Reduction =0.5753 LS2 -0.5703 LS + 1.003
Such that LS is the percent loss of support.

Both the Dijkstra and Markov Chains approaches allowed for the determination of a relationship between percent loss of support and average tie life. In addition, using these generated tie lives as a function of loss of adjacent tie support condition, equations for tie life reduction due to this loss of adjacent tie support were also generated.

[13] From Group F as shown in Figure 4.

GENERAL CONCLUSION

This report addresses the issue of tie life and the effect of adjacent poor condition ties on that life. Specifically, this report looks at the effect of poor adjacent tie condition on the life of a wood cross-tie. In this report, a series of analyses has been presented using tie condition data collected by the Aurora tie inspection system over a three-year period from 2016 through 2019. The analysis focused on the portion of the track where no tie gang or significant number of spot tie replacement occurred (about 40 miles and just under 100,000 cross-ties)

The ties were carefully aligned so as to accurately define the change in tie condition through the inspection years 2016, and 2019. Cross-correlation analysis was used to insure an accurate matching of the individual ties for approximately 40 miles of data over the three-year study period. These 40 miles represented track where it was determined that no major tie replacement activity, such as a system or regional tie gang, took place during the three-year study period.

After alignment, the study tie data set was divided into different tie support categories based on the condition of the adjacent cross-ties and the associated loss of support. Using Beam on Elastic Foundation (BOEF) theory, the percentage of load carried by these poor adjacent ties was calculated for each of the four categories, with category F serving as a baseline with all adjacent ties in good condition. Approximately 100,000 ties were included in this study. The tie condition inspection scores, which were subdivided into decimal subcategories, allowed for the calculation of a change in tie condition score for each tie, over the three-year period.

Following this further, the effect size analysis allowed for the quantification of the effect that the loss of adjacent tie support has on different tie groups. It was determined that the higher the loss of adjacent tie support the higher the effect size, and hence the higher the rate of tie degradation.

In addition, different surface fittings were performed in order to get an appropriate equation describing the behavior of the ties over time. Because of the complexity of the degradation behavior, different surface fitting equations were developed as a function of the initial tie conditions scores and the loss of adjacent tie support. Thus, different equations were developed to describe and predict the probability of a final score in 3 years given initial tie score and the associated loss of adjacent tie support. These equations considered the loss of adjacent tie support as a variable and the surface fittings were performed for each range of initial tie condition score independently.

The time variable was then introduced to the calculated probabilities. The degradation likelihood change over time was considered after modeling the tie score changes that happen within the 3-year data time period. The introduction of the time variable to the previous results was performed in two different ways:

- Exponential crosstie degradation over time
- Linear crosstie degradation over time

The study aimed at predicting the amount of time it will take for a “good tie” to have a high probability of failure (determined to be a 75%) based on its adjacent tie condition (loss of adjacent tie support).

The next level of analysis consisted of developing a life reduction factor based on loss of adjacent tie support. Three analysis approaches were used:

- A recursive function for tie life reconstruction
- A piecewise reconstruction of tie life as a function of varying support condition using Dijkstra’s algorithm.
- A Markov Chains analysis to predict the change in tie failure probability for different support conditions.

The results represented an extension of an earlier study where a simplified regression analysis approach was used to calculate tie life reduction. In addition to using a more accurate and effective modeling approach, this study also was able to calculate the average tie life as a function of support condition.

The recursive function made use of a heuristic method that confirmed the fact that loss of adjacent tie support contributes greatly to premature failure of the middle tie. As of the two other methods, the resulting life reduction factors, i.e. the loss of tie life as a function of support condition, based on both the Dijkstra’s (Method D) algorithm and Markov Chains (Method M) were less aggressive than the life reduction factor calculated using the original “simplified analysis” study (Method A). These two methods indicated that the effect of adjacent tie condition and associated loss of support is somewhat smaller than the original analysis, and in fact may be more realistic. The formula from Method A showed a dramatic reduction in tie life associated with modest loss of tie support, as compared to a more realistic reduction associated with the revised approaches.

Relationships were obtained for the tie life reduction factor as a function of percent loss of support as follows:

Dijkstra life reduction formula	Life Reduction = $1.005\ LS^2 - 1.013\ LS + 1.005$
Markov Chain Life reduction formula	Life Reduction =$0.5753\ LS^2 - 0.5703\ LS + 1.003$
Method A (Previous Study) life reduction formula	Life Reduction = $1.444\ LS^2 - 1.322\ LS + 0.9931$

However, despite their different degradation rates, all three methods confirm the fact that loss of adjacent tie support contributes to the premature degradation and failure of a tie. These findings can be useful to the railroad industry as they can provide a basis for more efficient maintenance and tie replacement planning.

RECOMMENDATIONS FOR FUTURE RESEARCH

While the analyses performed herein present important and practical results; the time span of available data was limited to three years. It is expected that the accuracy and effectiveness of the modeling would benefit from extending this research to a dataset with a longer time horizon and additional inspection cycles.

In addition, the study was limited to a data set of approximately 100,000 ties. Furthermore, the dataset was unbalanced, in that 80% of the ties belonged to the group where all the adjacent ties were in good condition. Future studies should aim to replicate these results on an even larger scale and longer time period. Extending this research to more miles of track with varying operating conditions, such as variations in annual MGT, variations in curvature/grade, and potential variations in climate can help to enhance the life reduction equation presented.

Finally, another limitation of the analysis was that the condition of the adjacent ties was held constant. This does not happen in the field and as such future research should consider the potential effects of changing adjacent tie support conditions over time.

ACKNOWLEDGEMENTS

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APPENDIX A

The figure below represents the bundle of A + B + C

A+B+C

A+B+C	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	342	375	219	154	362	166	74	59	95	148	44	116	32	15	15	14	16	8	17	12	13	2	2	7	2	7	2	4	2	2	20
1.1	-	538	329	233	611	295	159	174	188	258	75	234	49	23	29	15	26	19	9	13	22	5	5	6	5	6	3	4	5	6	24
1.2	-	-	230	168	359	219	103	125	190	234	83	214	57	21	32	16	11	10	19	15	17	7	4	6	2	3	1	1	1	3	26
1.3	-	-	-	140	302	178	88	116	156	235	81	181	39	20	31	25	11	11	16	20	16	7	5	1	1	4	3	2	6	3	29
1.4	-	-	-	-	240	154	76	107	165	222	99	200	52	10	22	13	13	15	8	19	19	6	5	1	1	1	2	2	1	-	19
1.5	-	-	-	-	-	120	65	88	91	200	58	162	44	22	24	16	19	6	9	25	19	5	1	7	-	4	-	3	1	-	17
1.6	-	-	-	-	-	-	30	49	42	95	39	165	44	13	40	28	16	9	12	19	18	3	1	2	2	1	1	1	1	4	10
1.7	-	-	-	-	-	-	-	45	54	119	24	163	64	23	51	16	18	36	12	16	18	7	1	2	-	2	1	1	1	-	12
1.8	-	-	-	-	-	-	-	-	35	110	27	157	52	36	42	39	23	18	19	21	36	5	2	2	2	1	-	-	-	5	12
1.9	-	-	-	-	-	-	-	-	-	141	72	189	96	38	33	24	35	25	28	43	44	13	4	-	6	3	5	-	-	1	18
2	-	-	-	-	-	-	-	-	-	-	43	106	30	30	16	25	25	28	24	29	32	4	3	4	-	1	1	1	2	3	14
2.1	-	-	-	-	-	-	-	-	-	-	-	160	92	29	56	27	44	46	59	86	134	9	13	7	8	6	7	1	5	7	20
2.2	-	-	-	-	-	-	-	-	-	-	-	-	39	26	33	24	22	48	37	61	143	7	3	5	5	2	4	6	4	1	20
2.3	-	-	-	-	-	-	-	-	-	-	-	-	-	6	27	29	21	13	26	35	125	4	5	-	5	4	4	4	7	7	16
2.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	13	15	14	17	16	30	124	8	7	-	3	-	3	2	3	5	28
2.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	8	17	11	32	56	7	7	3	2	3	1	5	4	2	8
2.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	8	7	10	19	47	4	2	2	1	6	2	-	8	3	11
2.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	7	17	56	6	3	2	2	4	2	5	3	3	10
2.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	13	76	8	4	3	6	1	2	3	-	7	20
2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	20	87	7	4	4	2	2	3	4	6	9	24
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	113	24	8	5	5	4	9	8	9	22	62
3.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	5	2	2	4	1	4	6	15
3.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	7	1	2	1	4	1	4	20
3.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	2	1	3	4	-	12
3.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	1	-	3	2	10	13
3.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	2	-	3	1	11
3.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-	1	2	5
3.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	2	2	9
3.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	3	10
3.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	15
4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	17

B+C

B+C	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	34	45	24	20	32	14	7	10	10	13	9	10	4	2	2	2	6	2	2	2	2	1		1		1		1	1		8
1.1		62	25	18	75	31	23	18	15	39	8	23	7	2	4	1	3	1	1	2	2		1	2		1	1	1	3	1	7
1.2			24	18	34	20	9	22	21	29	14	26	7	1	6	1	1		2	1	1			1			1			2	7
1.3				16	31	19	13	8	15	17	9	28	6	2	3	1	1		1	3	7	1			1		1	1	1	1	5
1.4					25	18	7	9	7	22	9	22	3			1	2		3	3		1		1	1		1				4
1.5						15	5	9	5	17	7	16	8	1	1	1	3	2	1	4		1				1					3
1.6							5	8	6	9	4	19	5	1	1			1	1	6	3	1				1					
1.7								5	6	12	1	16	10		5		3	1	1	2	5	1	1			1					1
1.8									1	15	7	16	3	8		9	1	2	5	1	2	1									3
1.9										20	6	18	7	6	6	2	9	3	2	4	6	4			1						1
2											6	11	3	3	2	3	3	4	4		5								1		5
2.1												26	8	5	3	2	3	7	7	13	14	2	1						1	2	4
2.2													2	4	2	2	2	4	3	6	14	1		1	1	1		1	2		1
2.3														1		4	2	1	3	6	17		2		1		1	1	3		1
2.4															2	2	2			2	8	1	2					1	2	2	2
2.5																	1	4	1	5	6	1	3						1		2
2.6																	1			2	7				1	1					1
2.7																			2	1	12	1							1	1	
2.8																			1	1	18	1								1	2
2.9																				4	12		1			1			1	2	2
3																					19	5	1		2		1	5	1	2	9
3.1																													1	1	3
3.2																								3		1					1
3.3																											1		1		2
3.4																										1		3		3	3
3.5																													1		2
3.6																															2
3.7																													1	1	3
3.8																															2
3.9																															3
4																															6

A+F

A+F	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	2,320	2,591	1,672	1,107	2,035	966	463	491	536	727	306	574	166	73	111	68	61	45	58	46	56	18	14	24	9	18	10	14	9	17	99
1.1	-	3,319	2,202	1,498	3,207	1,599	757	853	965	1,248	403	1,055	281	96	172	79	83	59	48	53	60	22	17	16	16	17	15	17	13	21	138
1.2	-	-	1,465	1,041	2,126	1,200	583	700	826	1,208	382	1,055	266	92	170	92	75	44	58	58	61	21	15	11	8	15	7	10	9	12	98
1.3	-	-	-	824	1,654	927	529	627	813	1,252	366	1,000	248	99	124	104	76	47	67	56	55	22	16	10	14	11	8	10	14	12	86
1.4	-	-	-	-	1,265	804	485	638	766	1,285	475	1,046	249	111	154	82	62	67	52	48	68	17	22	10	8	6	5	18	6	17	81
1.5	-	-	-	-	-	631	369	480	599	1,014	303	1,013	263	113	199	94	82	61	67	66	83	20	13	19	6	6	9	13	8	10	65
1.6	-	-	-	-	-	-	163	228	270	534	141	806	271	79	176	95	61	59	54	48	72	15	11	8	8	6	4	4	3	11	37
1.7	-	-	-	-	-	-	-	181	253	533	144	799	296	86	231	117	72	62	72	89	78	19	10	11	7	8	4	7	4	7	38
1.8	-	-	-	-	-	-	-	-	232	550	142	821	313	159	266	114	129	75	108	116	155	28	17	11	13	8	6	8	7	12	46
1.9	-	-	-	-	-	-	-	-	-	600	252	913	393	165	202	152	149	123	121	179	208	31	23	10	10	11	19	11	9	14	74
2	-	-	-	-	-	-	-	-	-	-	214	435	150	118	108	93	116	99	79	108	139	27	6	17	8	12	10	8	9	10	32
2.1	-	-	-	-	-	-	-	-	-	-	-	627	305	147	250	172	195	198	187	329	532	36	32	16	28	12	21	8	19	25	77
2.2	-	-	-	-	-	-	-	-	-	-	-	-	194	126	193	131	129	151	179	284	579	26	24	24	17	17	9	15	15	21	62
2.3	-	-	-	-	-	-	-	-	-	-	-	-	-	64	130	100	87	95	146	167	516	28	19	10	20	10	16	19	12	21	69
2.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	75	64	72	83	91	147	494	42	34	17	15	11	24	12	13	18	80
2.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	31	36	45	42	79	235	24	13	9	13	7	9	13	13	13	39
2.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	29	29	39	50	155	18	9	10	6	12	12	7	15	8	32
2.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	24	24	44	172	16	8	9	13	11	9	13	9	18	32
2.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	22	42	195	26	13	12	13	11	13	8	8	20	51
2.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	57	267	34	12	14	14	15	14	14	16	20	78
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	306	57	26	22	18	24	26	15	36	52	173
3.1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	4	9	6	8	7	5	3	15	29
3.2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	8	5	2	2	7	7	10	43
3.3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	3	9	2	6	5	3	34
3.4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	6	2	4	6	13	37
3.5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	5	4	6	4	25
3.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	2	3	6	12
3.7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	3	3	17
3.8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	8	18
3.9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	36
4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	38

A+B+C

A+B+C	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
1.0	14.6%	16.0%	9.3%	6.6%	15.4%	7.1%	3.1%	2.5%	4.1%	6.3%	1.9%	4.9%	1.4%	0.7%	0.7%	0.6%	0.7%	0.3%	0.7%	0.5%	0.6%	0.1%	0.1%	0.3%	0.1%	0.3%	0.1%	0.2%	0.1%	0.1%	0.8%
1.1	0.0%	16.0%	9.8%	6.9%	18.1%	8.8%	4.7%	5.2%	5.6%	7.7%	2.2%	6.9%	1.4%	0.7%	0.9%	0.4%	0.8%	0.6%	0.3%	0.4%	0.7%	0.1%	0.1%	0.2%	0.1%	0.2%	0.1%	0.1%	0.2%	0.2%	0.7%
1.2	0.0%	0.0%	10.6%	7.7%	16.5%	10.1%	4.7%	5.8%	8.7%	10.7%	3.8%	9.8%	2.6%	1.0%	1.5%	0.7%	0.5%	0.5%	0.9%	0.7%	0.8%	0.3%	0.2%	0.3%	0.1%	0.1%	0.1%	0.0%	0.0%	0.1%	1.2%
1.3	0.0%	0.0%	0.0%	8.1%	17.5%	10.3%	5.1%	6.7%	9.0%	13.6%	4.7%	10.5%	2.2%	1.2%	1.8%	1.5%	0.6%	0.6%	0.9%	1.2%	0.9%	0.4%	0.3%	0.1%	0.1%	0.2%	0.2%	0.1%	0.4%	0.2%	1.7%
1.4	0.0%	0.0%	0.0%	0.0%	16.3%	10.4%	5.1%	7.3%	11.2%	15.1%	6.7%	13.6%	3.6%	0.7%	1.5%	0.9%	0.9%	1.0%	0.6%	1.3%	1.3%	0.4%	0.3%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%	1.3%
1.5	0.0%	0.0%	0.0%	0.0%	0.0%	11.9%	6.4%	8.8%	9.0%	19.8%	5.7%	16.1%	4.4%	2.2%	2.4%	1.6%	1.9%	0.6%	0.9%	2.4%	1.9%	0.5%	0.1%	0.7%	0.0%	0.4%	0.0%	0.3%	0.1%	0.0%	1.7%
1.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.6%	7.6%	6.5%	14.8%	6.0%	25.6%	6.7%	2.0%	6.2%	4.3%	2.5%	1.4%	1.9%	3.0%	2.9%	0.5%	0.2%	0.3%	0.3%	0.2%	0.2%	0.2%	0.2%	0.6%	1.5%
1.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.7%	8.1%	17.8%	3.6%	24.4%	9.5%	3.5%	7.7%	2.4%	2.8%	2.4%	1.8%	2.5%	2.7%	1.1%	0.2%	0.3%	0.0%	0.3%	0.2%	0.2%	0.2%	0.0%	1.8%
1.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.4%	17.1%	4.2%	24.2%	8.1%	5.6%	6.5%	6.0%	3.6%	2.8%	2.9%	3.3%	5.6%	0.8%	0.3%	0.3%	0.3%	0.2%	0.0%	0.0%	0.0%	0.8%	1.9%
1.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	17.2%	8.8%	23.1%	11.7%	4.6%	4.0%	3.0%	4.3%	3.1%	3.5%	5.2%	5.4%	1.5%	0.5%	0.0%	0.8%	0.4%	0.6%	0.0%	0.0%	0.1%	2.2%
2.0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.3%	25.1%	7.0%	7.0%	3.9%	5.8%	6.0%	6.6%	5.8%	6.9%	7.6%	1.0%	0.7%	1.0%	0.0%	0.2%	0.2%	0.2%	0.5%	0.7%	3.4%
2.1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	19.5%	11.3%	3.5%	6.9%	3.3%	5.4%	5.6%	7.2%	10.5%	16.4%	1.1%	1.6%	0.9%	1.0%	0.7%	0.9%	0.1%	0.6%	0.9%	2.4%
2.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.0%	5.2%	6.8%	4.9%	4.6%	9.7%	7.5%	12.4%	29.1%	1.5%	0.6%	1.0%	1.0%	0.5%	0.8%	1.2%	0.9%	0.2%	4.1%
2.3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.8%	8.0%	8.5%	6.3%	3.9%	7.6%	10.5%	37.0%	1.2%	1.3%	0.0%	1.5%	1.2%	1.2%	1.2%	2.0%	2.1%	4.8%
2.4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	5.3%	4.8%	5.9%	5.6%	10.3%	43.2%	2.8%	2.3%	0.0%	1.0%	0.0%	1.0%	0.7%	0.9%	1.6%	9.9%
2.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.5%	4.7%	10.0%	6.4%	18.7%	32.1%	4.1%	4.3%	1.7%	1.2%	1.7%	0.6%	2.9%	2.4%	1.2%	4.8%
2.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.3%	5.4%	7.7%	14.4%	36.0%	3.1%	1.5%	1.5%	0.9%	4.7%	1.5%	0.0%	6.2%	2.3%	8.6%
2.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.2%	5.8%	13.7%	44.9%	4.9%	2.4%	1.6%	1.6%	3.2%	1.6%	4.0%	2.5%	2.5%	8.0%
2.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	8.8%	51.0%	5.6%	2.7%	2.0%	4.0%	0.7%	1.3%	2.0%	0.0%	4.8%	13.6%
2.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.4%	50.4%	4.0%	2.4%	2.3%	1.2%	1.2%	1.7%	2.3%	3.7%	5.4%	14.0%
3.0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	42.0%	8.8%	3.0%	1.9%	1.9%	1.5%	3.4%	2.8%	3.4%	8.3%	23.0%
3.1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.5%	0.0%	12.4%	5.0%	5.0%	9.9%	2.5%	10.3%	15.3%	37.2%
3.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	9.0%	15.8%	2.3%	4.8%	2.3%	9.0%	2.3%	9.0%	45.5%
3.3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.2%	0.0%	8.2%	4.1%	12.4%	17.1%	0.0%	50.0%
3.4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	9.6%	3.7%	0.0%	8.1%	6.4%	31.3%	40.9%
3.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.4%	10.4%	0.0%	15.7%	5.2%	58.2%
3.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.8%	0.0%	10.8%	21.5%	56.9%
3.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.9%	13.9%	13.9%	65.4%
3.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.1%	21.2%	71.7%
3.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.1%	93.9%
4.0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%

F+A

A+F	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	15.8%	17.6%	11.4%	7.5%	13.8%	6.6%	3.1%	3.3%	3.6%	4.9%	2.1%	3.9%	1.1%	0.5%	0.8%	0.5%	0.4%	0.3%	0.4%	0.3%	0.4%	0.1%	0.1%	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.7%
1.1	0.0%	18.1%	12.0%	8.2%	17.5%	8.7%	4.1%	4.7%	5.3%	6.8%	2.2%	5.8%	1.5%	0.5%	0.9%	0.4%	0.5%	0.3%	0.3%	0.3%	0.3%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.8%
1.2	0.0%	0.0%	12.5%	8.9%	18.2%	10.2%	5.0%	6.0%	7.1%	10.3%	3.3%	9.0%	2.3%	0.8%	1.5%	0.8%	0.6%	0.4%	0.5%	0.5%	0.5%	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.8%
1.3	0.0%	0.0%	0.0%	9.1%	18.2%	10.2%	5.8%	6.9%	9.0%	13.8%	4.0%	11.0%	2.7%	1.1%	1.4%	1.1%	0.8%	0.5%	0.7%	0.6%	0.6%	0.2%	0.2%	0.1%	0.2%	0.1%	0.1%	0.1%	0.2%	0.1%	0.9%
1.4	0.0%	0.0%	0.0%	0.0%	16.1%	10.2%	6.2%	8.1%	9.8%	16.4%	6.1%	13.3%	3.2%	1.4%	2.0%	1.0%	0.8%	0.9%	0.7%	0.6%	0.9%	0.2%	0.3%	0.1%	0.1%	0.1%	0.1%	0.2%	0.1%	0.2%	1.0%
1.5	0.0%	0.0%	0.0%	0.0%	0.0%	11.3%	6.6%	8.6%	10.7%	18.1%	5.4%	18.1%	4.7%	2.0%	3.5%	1.7%	1.5%	1.1%	1.2%	1.2%	1.5%	0.4%	0.2%	0.3%	0.1%	0.1%	0.2%	0.2%	0.1%	0.2%	1.2%
1.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.2%	7.2%	8.5%	16.9%	4.5%	25.5%	8.6%	2.5%	5.6%	3.0%	1.9%	1.9%	1.7%	1.5%	2.3%	0.5%	0.3%	0.3%	0.3%	0.2%	0.1%	0.1%	0.1%	0.3%	1.2%
1.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.8%	8.1%	17.0%	4.6%	25.5%	9.5%	2.7%	7.4%	3.7%	2.3%	2.0%	2.3%	2.8%	2.5%	0.6%	0.3%	0.4%	0.2%	0.3%	0.1%	0.2%	0.1%	0.2%	1.2%
1.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.0%	16.5%	4.3%	24.6%	9.4%	4.8%	8.0%	3.4%	3.9%	2.2%	3.2%	3.5%	4.6%	0.8%	0.5%	0.3%	0.4%	0.2%	0.2%	0.2%	0.2%	0.4%	1.4%
1.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	16.4%	6.9%	24.9%	10.7%	4.5%	5.5%	4.1%	4.1%	3.4%	3.3%	4.9%	5.5%	0.8%	0.6%	0.3%	0.3%	0.3%	0.5%	0.3%	0.2%	0.4%	2.0%
2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.9%	24.3%	8.4%	6.6%	6.0%	5.2%	6.5%	5.5%	4.4%	5.7%	7.8%	1.5%	0.3%	0.9%	0.4%	0.7%	0.6%	0.4%	0.5%	0.6%	1.8%
2.1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	19.5%	9.5%	4.6%	7.8%	5.3%	6.1%	6.2%	5.8%	10.2%	16.5%	1.1%	1.0%	0.5%	0.9%	0.4%	0.7%	0.2%	0.6%	0.8%	2.4%
2.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.8%	5.7%	8.8%	6.0%	5.9%	6.9%	8.2%	12.9%	26.4%	1.2%	1.1%	1.1%	0.8%	0.8%	0.4%	0.7%	0.7%	1.0%	2.8%
2.3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.2%	8.5%	6.5%	5.7%	6.2%	9.5%	10.9%	33.7%	1.8%	1.2%	0.7%	1.3%	0.7%	1.0%	1.2%	0.8%	1.4%	4.5%
2.4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.8%	5.0%	5.6%	6.4%	7.0%	11.4%	38.2%	3.3%	2.6%	1.3%	1.2%	0.9%	1.9%	0.9%	1.0%	1.4%	6.2%
2.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.0%	5.8%	7.2%	6.8%	12.7%	37.8%	3.9%	2.1%	1.4%	2.1%	1.1%	1.4%	2.1%	2.1%	2.1%	6.3%
2.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.7%	6.7%	9.0%	11.6%	36.0%	4.2%	2.1%	2.3%	1.4%	2.8%	2.8%	1.6%	3.5%	1.9%	7.4%
2.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.0%	6.0%	10.9%	42.8%	4.0%	2.0%	2.2%	3.2%	2.7%	2.2%	3.2%	2.2%	4.5%	8.0%
2.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.1%	9.7%	44.9%	6.0%	3.0%	2.8%	3.0%	2.5%	3.0%	1.8%	1.8%	4.6%	11.8%
2.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.3%	48.1%	6.1%	2.2%	2.5%	2.5%	2.7%	2.5%	2.5%	2.9%	3.6%	14.1%
3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	41.3%	7.5%	3.4%	2.9%	2.4%	3.1%	3.4%	2.0%	4.7%	6.8%	22.6%
3.1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.3%	4.5%	10.2%	6.8%	9.1%	8.0%	5.7%	3.4%	17.0%	33.0%
3.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.6%	9.0%	5.6%	2.2%	2.2%	7.9%	7.9%	11.2%	48.3%
3.3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.5%	4.5%	13.4%	3.0%	9.0%	7.5%	4.5%	50.7%
3.4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.6%	8.3%	2.8%	5.6%	8.3%	18.1%	51.4%
3.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.3%	10.4%	8.3%	12.5%	8.3%	52.1%
3.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	11.5%	7.7%	11.5%	23.1%	46.2%
3.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	14.8%	11.1%	11.1%	63.0%
3.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.1%	28.6%	64.3%
3.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	14.3%	85.7%
4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%

B+C

B+C	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3	3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
1	12.7%	16.7%	8.8%	7.5%	12.1%	5.4%	2.5%	3.7%	3.8%	5.0%	3.4%	3.7%	1.7%	0.9%	0.9%	0.8%	2.1%	0.8%	0.9%	0.8%	0.8%	0.4%	0.0%	0.4%	0.0%	0.4%	0.0%	0.4%	0.4%	0.0%	2.9%
1.1	0.0%	16.3%	6.7%	4.7%	19.8%	8.2%	6.1%	4.7%	4.1%	10.3%	2.1%	6.1%	1.7%	0.5%	1.2%	0.3%	0.9%	0.3%	0.3%	0.5%	0.6%	0.0%	0.3%	0.6%	0.0%	0.3%	0.3%	0.3%	0.9%	0.3%	1.8%
1.2	0.0%	0.0%	9.6%	7.2%	13.8%	8.2%	3.6%	9.0%	8.6%	11.5%	5.4%	10.6%	2.7%	0.4%	2.3%	0.4%	0.5%	0.0%	0.9%	0.5%	0.4%	0.0%	0.0%	0.5%	0.0%	0.0%	0.5%	0.0%	0.0%	0.9%	2.7%
1.3	0.0%	0.0%	0.0%	8.2%	16.4%	9.9%	7.0%	4.2%	7.6%	8.8%	4.7%	14.5%	2.9%	1.2%	1.7%	0.6%	0.6%	0.0%	0.6%	1.7%	3.5%	0.6%	0.0%	0.0%	0.6%	0.0%	0.6%	0.6%	0.6%	0.6%	2.4%
1.4	0.0%	0.0%	0.0%	0.0%	17.5%	12.7%	4.7%	6.4%	4.7%	16.0%	6.5%	15.8%	2.4%	0.0%	0.0%	0.8%	1.6%	0.0%	2.4%	2.3%	0.0%	0.8%	0.0%	0.8%	0.8%	0.0%	0.8%	0.0%	0.0%	0.0%	3.1%
1.5	0.0%	0.0%	0.0%	0.0%	0.0%	14.9%	4.5%	9.2%	4.6%	16.2%	6.7%	16.1%	8.3%	1.1%	1.1%	1.2%	3.4%	2.2%	1.1%	3.6%	0.0%	1.2%	0.0%	0.0%	0.0%	1.1%	0.0%	0.0%	0.0%	0.0%	3.4%
1.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.5%	11.4%	8.3%	12.9%	5.0%	26.6%	6.3%	1.7%	1.6%	0.0%	0.0%	1.6%	1.7%	8.5%	4.8%	1.6%	0.0%	0.0%	0.0%	1.6%	0.0%	0.0%	0.0%	0.0%	0.0%
1.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.8%	8.0%	16.6%	1.8%	22.0%	13.4%	0.0%	7.0%	0.0%	4.8%	1.6%	1.6%	3.4%	6.6%	1.6%	1.6%	0.0%	0.0%	1.6%	0.0%	0.0%	0.0%	0.0%	1.8%
1.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.5%	20.1%	9.4%	20.4%	4.5%	11.1%	0.0%	11.4%	1.7%	3.0%	6.0%	1.5%	3.2%	1.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.5%
1.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	21.0%	6.0%	18.6%	7.4%	6.2%	6.1%	2.5%	9.8%	3.6%	2.5%	3.7%	6.1%	3.7%	0.0%	0.0%	1.3%	0.0%	0.0%	0.0%	0.0%	0.0%	1.2%
2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	13.1%	21.3%	5.1%	5.1%	4.6%	5.1%	6.9%	7.4%	8.5%	0.0%	9.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.3%	0.0%	10.8%
2.1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	26.0%	8.2%	4.7%	3.5%	2.4%	3.5%	7.1%	7.0%	12.9%	14.1%	2.4%	1.2%	0.0%	0.0%	0.0%	0.0%	0.0%	1.2%	2.3%	3.6%
2.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.1%	7.4%	4.6%	4.6%	5.1%	7.4%	5.5%	12.0%	28.5%	2.3%	0.0%	2.3%	2.3%	2.7%	0.0%	2.3%	5.1%	0.0%	2.7%
2.3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	0.0%	8.8%	5.3%	2.7%	6.1%	14.9%	39.4%	0.0%	3.5%	0.0%	2.7%	0.0%	2.7%	2.7%	6.1%	0.0%	2.7%
2.4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.7%	9.5%	6.7%	0.0%	0.0%	6.7%	31.4%	4.8%	6.7%	0.0%	0.0%	0.0%	0.0%	4.8%	6.7%	6.7%	9.5%
2.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	16.7%	4.4%	21.1%	22.2%	4.4%	13.3%	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	0.0%	8.9%
2.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.8%	0.0%	0.0%	12.8%	52.0%	0.0%	0.0%	0.0%	8.8%	8.8%	0.0%	0.0%	0.0%	0.0%	8.8%
2.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	12.1%	6.1%	63.6%	6.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.1%	6.1%	0.0%
2.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.5%	4.5%	72.0%	5.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.5%	9.1%
2.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	15.3%	50.5%	0.0%	4.7%	0.0%	0.0%	4.7%	0.0%	0.0%	5.8%	9.5%	9.5%
3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	42.4%	10.4%	2.6%	0.0%	4.9%	0.0%	2.6%	10.4%	2.6%	4.9%	19.4%
3.1	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	21.6%	21.6%	56.7%
3.2	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	56.7%	0.0%	21.6%	0.0%	0.0%	0.0%	0.0%	21.6%
3.3	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	23.3%	0.0%	26.7%	0.0%	50.0%
3.4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	12.3%	0.0%	27.4%	0.0%	30.1%	30.1%
3.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	31.8%	0.0%	68.2%
3.6	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%
3.7	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	18.4%	18.4%	63.2%
3.8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%
3.9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%
4	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100.0%

APPENDIX B

INTRODUCING THE TIME VARIABLE TO THE PROBABILITY OF TIE DEGRADATION - EXPONENTIAL

Exponential Degradation of Wood Ties

The tie condition or Tie Score can be modeled by the following equation:

$$Tie\ Score = A\, e^{B\, t}$$

Such that A and B are constants and t is time.

The initial score at t= 0 is 1, so:

$$when\ t = 0: \ \ => \ \ Tie\ Score = 1 => A = 1$$

T is dependent on the different loss of support conditions, and T(Ls) represents the average tie life as computed by the formula developed in Chapter 7 (equation 5):

$$T(Ls) = (1.444\ Ls^2 - 1.322\ Ls + 0.9931) * T$$

$$Tie\ Score(t = T(Ls)) = \ e^{B * T(Ls)} = 4$$

$$B = \frac{\ln(4)}{T(Ls)}$$

$$\text{Tie Score } = 4^{\left({}^{t}/_{T(Ls)}\right)} \qquad \text{Equation 3}$$

where:

- $T\ (Ls) = (1.444\ Ls^2 - 1.322\ Ls + 0.9931) * T$
- And t is the time in years.

∆T: Time to go from a score SI (initial) to a score SF (final)

Using Equation 23, SI (Initial Tie Score) and SF (Final Tie Score) are expressed as follow:

$$SI = 4^{\left({}^{tSI}/_{T(Ls)}\right)}$$

$$SF = 4^{\left({}^{tSF}/_{T(Ls)}\right)}$$

where tSI and tSF represent the time for a tie to reach score SI and Score SF respectively.
And T(Ls) is the average tie life (based on the loss of support).

$$tSI = \frac{\ln(SI)}{\ln(4)} * T(Ls)$$
$$tSF = \frac{\ln(SF)}{\ln(4)} * T(Ls)$$

$$\Delta T = tSF - tSI = \frac{T(Ls)}{\ln(4)} * \ln(\frac{Score\ SF}{Score\ SI})$$

So, by replacing T(Ls) by its expression, the time to go from a score SI (initial) to a score SF (final) is:

$$\Delta T = \frac{(1.444\ Ls2\ -\ 1.322\ Ls\ +\ 0.9931) * T}{\ln(4)} * \ln(\frac{SF}{SI})$$ Equation 13

Exponential Increase of Probability

Assuming that the probability is exponentially increasing over time, the change of probability over time can be described as:

$$\mathrm{Probability(t)} \ = \ \mathrm{C\ e^{kt}}$$ Equation AB1

Such that C and k are constants.

Determining the constants C and k

When t = 3 years*:* **Probability(t=3) = P (SI, Ls, SF)**

Where P(SI, Ls,SF) are the equations developed.

$$P(SI, Ls, SF) = C\ e^{3k}$$

$$\mathrm{C} = \frac{\mathrm{P(SI, Ls, SF)}}{\mathrm{e^{3k}}}$$ Equation AB2

Note:

- The probability to change scores from SI to SF is referred to as Tr1 (Threshold probability)
- **ΔT= tSF- tSI** is the amount of time for the tie score to move from SI to SF, as expressed in Equation 4.

When **t = ΔT years ,**

Tr1 = Probability(t=ΔT)

$$\mathrm{Tr1} = \frac{\mathrm{P(SI, Ls, SF)}}{e^{3k}} * e^{\,k\Delta T}$$

$$\frac{\mathrm{Tr1}}{\mathrm{P(SI, Ls, SF)}} = e^{k\Delta T - 3k}$$

$$k = \frac{\ln(\frac{\mathrm{Tr1}}{\mathrm{P(SI, Ls, SF)}})}{\Delta T - 3}$$ Equation AB3

From Equation AB3:

$$3k = \frac{3 * \ln(\frac{\mathrm{Tr1}}{\mathrm{P(SI, Ls, SF)}})}{\Delta T - 3}$$

$$e^{3k} = e^{\frac{3 * \ln(\frac{\mathrm{Tr1}}{\mathrm{P(SI,Ls,SF)}})}{\Delta T - 3}}$$

$$e^{3k} = \left(\frac{\mathrm{Tr1}}{\mathrm{P(SI,Ls,SF)}}\right)^{\left(\frac{3}{\Delta T - 3}\right)}$$ Equation AB4

From Equation AB2:

$$C = \frac{\mathrm{P(SI, Ls, SF)}}{(\frac{\mathrm{Tr1}}{\mathrm{P(SI, Ls, SF)}})^{(\frac{3}{\Delta T - 3})}}$$

$$C = \mathrm{P(SI, Ls, SF)} * \ (\frac{\mathrm{Tr1}}{\mathrm{P(SI, Ls, SF)}})^{-(\frac{3}{\Delta T - 3})}$$

$$C = \mathrm{P(SI, Ls, SF)} * \ \left(\frac{\mathrm{Tr1}}{\mathrm{P(SI, Ls, SF)}}\right)^{\left(\frac{3}{3 - \Delta T}\right)}$$

$$C = \mathrm{P(SI, Ls, SF)} * (\mathrm{P(SI, Ls, SF)})^{(\frac{3}{\Delta T - 3})} * \ (\mathrm{Tr1})^{(\frac{3}{3 - \Delta T})}$$

$$C = (\mathrm{P(SI, Ls, SF)})^{(1 + \frac{3}{\Delta T - 3})} * \ (\mathrm{Tr1})^{(\frac{3}{3 - \Delta T})}$$

$$C = (\mathrm{P(SI, Ls, SF)})^{(\frac{\Delta T}{\Delta T - 3})} * \ (\mathrm{Tr1})^{(\frac{3}{3 - \Delta T})}$$ Equation AB5

From Equation AB1, AB4, and AB5:

$$\mathrm{Probability(t)} \ = \ (\mathrm{P(SI, Ls, SF)})^{(\frac{\Delta T}{\Delta T - 3})} * \ (\mathrm{Tr1})^{(\frac{3}{3 - \Delta T})} \ e^{\frac{\ln(\frac{\mathrm{Tr1}}{\mathrm{P(SI,Ls,SF)}})}{\Delta T - 3} t}$$ *Equation 5*

Determining t(Tr1) time it takes for the probability to be higher than threshold Tr1

Let t(Tr2) be the required time for the probability P(SI, Ls,SF) to be higher than the Threshold probability (Tr2), such as:

$$\mathbf{Tr2} = \mathbf{Tr1}^{(\mathbf{0.5}/(\mathbf{SF}-\mathbf{SI}-\mathbf{0.5}))} \qquad \textit{Equation AB6}$$

From Equation 5**Error! Reference source not found.**

$$\text{Probability}(t(\text{Tr2})) = \text{Tr2} = \text{C}\, e^{k\, t(\text{Tr2})}$$

$$t(\text{Tr2}) = \frac{\ln(\frac{\text{Tr2}}{\text{C}})}{k}$$

replacing with k and C with their respective the expressions from Equation AB2 and AB3

$$t = \frac{\ln\left(\frac{\text{Tr2}}{\left(P(\text{SI}, \text{Ls}, \text{SF})\right)^{\left(\frac{\Delta T}{\Delta T-3}\right)} * \ (\text{Tr1})^{\left(\frac{3}{3-\Delta T}\right)}} \right)}{\frac{\ln(\frac{\text{Tr1}}{P(\text{SI}, \text{Ls}, \text{SF})})}{\Delta T - 3}}$$

$$t(\text{Tr2}) = (\Delta T - 3) * \frac{\ln\left(\frac{\text{Tr2}}{\left(P(\text{SI}, \text{Ls}, \text{SF})\right)^{\left(\frac{\Delta T}{\Delta T-3}\right)} * \ (\text{Tr1})^{\left(\frac{3}{3-\Delta T}\right)}} \right)}{\ln\left(\frac{\text{Tr1}}{P(\text{SI}, \text{Ls}, \text{SF})} \right)}$$

from Equation AB6:

$$t(\text{Tr1}) = (\Delta T - 3) * \frac{\ln\left(\frac{\text{Tr1}^{\left(\frac{0.5}{\text{SF}-\text{SI}-0.5}\right)}}{P(\text{SI}, \text{Ls}, \text{SF})^{\left(\frac{\Delta T}{\Delta T-3}\right)} * \ \text{Tr1}^{\left(\frac{3}{3-\Delta T}\right)}} \right)}{\ln\left(\frac{\text{Tr1}}{P(\text{SI}, \text{Ls}, \text{SF})} \right)}$$

$$t(\text{Tr1}) = (\Delta T - 3) * \frac{\ln\left(\frac{\text{Tr1}^{\left(\frac{0.5}{\text{SF}-\text{SI}-0.5}\right)}\ \text{Tr1}^{\left(\frac{3}{\Delta T-3}\right)}}{P(\text{SI}, \text{Ls}, \text{SF})^{\left(\frac{\Delta T}{\Delta T-3}\right)}} \right)}{\ln\left(\frac{\text{Tr1}}{P(\text{SI}, \text{Ls}, \text{SF})} \right)}$$

$$t(\mathrm{Tr}) = (\Delta \mathrm{T} - 3) * \frac{\ln\left(\dfrac{\mathrm{Tr1}^{\left(\frac{0.5}{\mathrm{SF}-\mathrm{SI}-0.5} * \frac{3}{\Delta \mathrm{T}-3}\right)}}{\mathrm{P}(\mathrm{SI}, \mathrm{Ls}, \mathrm{SF})^{\left(\frac{\Delta T}{\Delta \mathrm{T}-3}\right)}} \right)}{\ln\left(\dfrac{\mathrm{Tr1}}{\mathrm{P}(\mathrm{SI}, \mathrm{Ls}, \mathrm{SF})} \right)}$$

Replacing ΔT with its expression in Equation 4 gives Equation 6.

So, the time it takes for the probability of a tie to move from score SI to a final Score SF to be higher than threshold Tr1 is expressed by the following Equation:

$\mathbf{t(Tr)} = \left(\frac{(1.444\,Ls2\ -\ 1.322\,Ls\ +\ 0.9931) * T}{\ln(4)} * \ln\left(\frac{SF}{SI}\right) - 3\right) * \frac{\ln\left(\dfrac{\mathrm{Tr1}\ \wedge\left(\frac{0.5}{\mathrm{SF}-\mathrm{SI}-0.5} * \frac{3}{\frac{(1.444\,Ls2 - 1.322\,Ls + 0.9931)*T}{\ln(4)} * \ln(\frac{SF}{SI}) - 3}\right)}{\mathrm{P}(\mathrm{SI}, \mathrm{Ls}, \mathrm{SF})\wedge\left(\frac{\frac{(1.444\,Ls2 - 1.322\,Ls + 0.9931)*T}{\ln(4)} * \ln(\frac{SF}{SI})}{\frac{(1.444\,Ls2 - 1.322\,Ls + 0.9931)*T}{\ln(4)} * \ln(\frac{SF}{SI}) - 3}\right)} \right)}{\ln\left(\dfrac{\mathrm{Tr1}}{\mathrm{P}(\mathrm{SI}, \mathrm{Ls}, \mathrm{SF})} \right)}$	Equation 6

where:

- Ls: Loss of support
- Tr1: The threshold set
- T: Average tie life
- SI: Initial Score
- SF: Final Score
- P(SI, Ls, SF): the probability for a tie with an initial Score SI to reach a final score SF in 3 years, from the modelling presented in the previous section.

APPENDIX C

INTRODUCING THE TIME VARIABLE TO THE PROBABILITY OF TIE DEGRADATION - LINEAR

Linear Degradation of Wood Ties

The Tie Score (tie condition) is then modelled by the following equation:

$$Tie\ Score = A * t + B$$

where A and B are constants and t is time.

The initial score at t = 0 is 1, so:

$$when\ t = 1:\ \ => \ \ Tie\ Score = 1 => A = \frac{3}{T-1}$$

As shown in Chapter 6, T is affected by the different loss of support conditions, and T(Ls) represents the average tie life as computed by the formula developed in Reference 9:

$$\mathrm{T(Ls)} = (1.444\ \mathrm{Ls}^2 - 1.322\ \mathrm{Ls} + 0.9931) * \mathrm{T}$$

$$Tie\ Score(t = T(Ls)) = \frac{3}{T(Ls)} * T(Ls) + B = 4$$

$$B = \frac{\mathrm{T(Ls)} - 4}{T(Ls) - 1}$$

So,

$$Tie\ Score\ (t) = 1 + \frac{3t - 1}{T(Ls) - 1} \qquad \text{Equation 7}$$

where,

$$\mathrm{T\ (Ls)} = (1.444\ \mathrm{Ls}^2 - 1.322\ \mathrm{Ls} + 0.9931) * \mathrm{T},$$

T is the input average tie life, and t is the time in years.

ΔT: Time to go from a score SI (initial) to a score SF (final)

From Equation 7,

$$\text{Tie Score SI} = 1 + \frac{3 *\ tSI - 1}{T(Ls) - 1}$$

$$\text{Tie Score SF} = 1 + \frac{3 *\ tSF - 1}{T(Ls) - 1}$$

Such that tSI and tSF represent respectively the time for a tie to reach score SI and Score SF, respectively.

And T(Ls) is the average tie life (based on the loss of support).

$$tSI = \frac{SI * (T(Ls) - 1) + 4 - T}{3}$$
$$tSF = \frac{SF * (T(Ls) - 1) + 4 - T}{3}$$

So, the time to go from a score SI (initial) to a score SF (final) is:

$\Delta T = tSF - tSI = \frac{S(F - FI) * (T(Ls) - 1)}{3}$	Equation 8

Linear Growth of Probability (Over Time)

Assuming that the probability is linearly growing over time, the change of probability over time can be described as:

$$Probability(t) = A * t + B$$

Such that A and B are constants.

When t = 3 years*:* **Probability(t=3) = P (SI, Ls, SF)**

$$P(SI, Ls, SF) = 3A + B$$

When **t = ΔT years, (ΔT=tSF-tSI)** (i.e., the amount of time for the tie score to move from SI to SF), the probability to change scores from SI to SF is referred to as Tr (Threshold probability)

$$\text{Probability } (t=\Delta T) = Tr = A * \Delta T + B$$

$$A = \frac{P(SI, Ls, SF) - Tr}{3 - \Delta T}$$

$$B = \frac{3 * Tr - \Delta T * P(SI, Ls, SF)}{3 - \Delta T}$$

$\text{Probability}(t) = \frac{P(SI, Ls, SF) - Tr}{3 - \Delta T} * t + \frac{3 * Tr - \Delta T * P(SI, Ls, SF)}{3 - \Delta T}$	Equation 9

Similarly to the previous section Let t(Tr2) be the required time for the probability P(SI, Ls,SF) to be higher than the Threshold probability (Tr2).

$$t = \left(\text{Probability}(t) - \frac{3 * \text{Tr} - \Delta T * P(SI, Ls, SF)}{3 - \Delta T}\right) * \frac{3 - \Delta T}{P(SI, Ls, SF) - \text{Tr}}$$

$$t = \left(\text{Tr2} - \frac{3 * \text{Tr} - \Delta T * P(SI, Ls, SF)}{3 - \Delta T}\right) * \frac{3 - \Delta T}{P(SI, Ls, SF) - \text{Tr}}$$

$$t = \left(\frac{\text{Tr2} * (3 - \Delta T) - 3 * \text{Tr} + \Delta T * P(SI, Ls, SF)}{P(SI, Ls, SF) - \text{Tr}}\right)$$

$$t = \frac{\Delta T * (P(SI, Ls, SF) - \text{Tr2}) + 3 * (\text{Tr2} - \text{Tr})}{P(SI, Ls, SF) - \text{Tr}}$$

$$t = \frac{\Delta T * \left(P(SI, Ls, SF) - \text{Tr}^{\left(\frac{0.5}{SF - SI - 0.5}\right)}\right) + 3 * (\text{Tr}^{\left(\frac{0.5}{SF - SI - 0.5}\right)} - \text{Tr})}{P(SI, Ls, SF) - \text{Tr}}$$

So, the time it takes for the probability of a tie to move from score SI to a final Score SF to be higher than threshold Tr is expressed by the following Equation:

$$t = \frac{\frac{S(F - FI) * \left((1.444\, Ls2 - 1.322\, Ls + 0.9931) * T - 1\right)}{3} * \left(P(SI, Ls, SF) - \text{Tr}^{\left(\frac{0.5}{SF - SI - 0.5}\right)}\right) + 3 * (\text{Tr}^{\left(\frac{0.5}{SF - SI - 0.5}\right)} - \text{Tr})}{P(SI, Ls, SF) - \text{Tr}} \quad \text{Equation 10}$$

where:

- Ls: Loss of support
- Tr: The threshold set
- T: Average tie life
- SI: Initial Score
- SF: Final Score
- P (SI, Ls, SF): the probability for a tie with an initial Score SI to reach a final score SF in 3 years, from the modelling presented in the previous section.

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